

Problems in Physics

100+ Solved Problems on

***Dual Nature of
Radiation and Matter***

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Preface

We collected and solved 120 good problems on photoelectric effect and wave-particle duality for Einstein's Noble Prize Centenary Year Celebrations (ENOPCYC).

This book provides a collection of good problems and their solutions. We have tried to select questions that improves your problem solving skills, test your conceptual understanding, and help you in exam preparation.

The book also covers relevant concepts, in brief. These are enough to solve problems given in this book.

There are separate chapters on questions, answers and solutions. The questions are arranged in increasing order of complexity. The multiple choice questions are of single option correct or multiple options correct type.

Solution are provided in the last chapter. If you can't solve a problem, give it a retry before referring to the solution. This will help you identify the critical points in the problems, which in turn, will accelerate the learning process. To us, every problem, is a valuable resource to unravel a deeper understanding of the underlying physical concepts. We believe that getting the right answer is often not as important as the process followed to arrive at it.

If a student seriously attempts all the problems in this book, he/she will naturally develop the ability to analyze and solve complex problems in a simple and logical manner using a few, well-understood principles.

We would be glad to hear from you for any suggestions/corrections on the improvement of the book.

This work would not have been possible without the constant support of our wives Reena and Nandini and children Akshaj, Viraj and Maitreyi.

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Contents

Contents	v
1 Key Concepts	1
1.1 Radiations	1
1.2 Einstein's Equation	2
1.3 de Broglie Wavelength	3
2 Questions and Solutions	4

1.1 Radiations

Energy flux S is the rate of transfer of energy through a surface. Let a plane wave is moving in a direction \hat{k} and a surface of area A is placed normal to its direction of motion. The energy flowing through this surface per unit time (i.e., Power P) is given by

$$P = SA. \quad (1)$$

Energy flux and intensity are often used interchangeably.

The frequency ν of radiation is related to its wavelength λ by

$$\nu\lambda = c, \quad (2)$$

where $c = 3 \times 10^8$ m/s is the speed of light.

The radiation can be viewed as a stream of photons moving with a speed c in the direction \hat{k} . The energy of a photon of radiation frequency ν is given by

$$E = h\nu, \quad (3)$$

where $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$ is Planck's constant. A convenient unit of photon's energy is electronvolt (eV). To convert energy from joule to electronvolt, divide it by electron's charge $e = 1.6 \times 10^{-19}$ coulomb. It is worth memorizing $hc = 1240 \text{ nm}\cdot\text{eV}$ which gives

$$E = \frac{hc}{\lambda} = \frac{1240}{\lambda \text{ (in nm)}} \text{ eV.} \quad (4)$$

Photons flux n is the number of photons crossing a unit area in a unit time. It is related to the energy-flux by

$$n = S/E. \quad (5)$$

The linear momentum of a photon of radiation wavelength λ is given by

$$p = h/\lambda. \quad (6)$$

1.2 Einstein's Equation

In photoelectric effect, a photon of energy $h\nu$ gives its entire energy to an electron. The electron comes out of the metal after doing a work ϕ against the binding force of the metal (ϕ is called work function). It also loses some energy due to collision with atoms etc. The maximum kinetic energy of the ejected photoelectrons is given by

$$K_{\max} = h\nu - \phi. \quad (7)$$

The cut-off (or threshold) frequency ν_c is the frequency at which $K_{\max} = 0$. The radiation of frequency less

than ν_c cannot eject photoelectrons. Its value is given by

$$\nu_c = \phi/h. \quad (8)$$

The stopping potential is the negative potential applied to the collector to stop photocurrent. It is given by

$$V_0 = K_{\max}/e. \quad (9)$$

1.3 de Broglie Wavelength

The de Broglie wavelength of a particle of momentum p is given by

$$\lambda = h/p. \quad (10)$$

If a particle moves with a relativistic speed then its energy E is related to momentum p by

$$E^2 = (pc)^2 + (mc^2)^2, \quad (11)$$

where m is the rest mass of the particle and mc^2 is its rest mass energy.

Questions and Solutions

Radiation Energy

Q 1. A 2 mW laser operates at wavelength of 500 nm. The number of photons that will be emitted per second is [Given $h = 6.6 \times 10^{-34}$ J·s and $c = 3.0 \times 10^8$ m/s.]
(2019 m)

- (A) 5×10^{15} (B) 1.5×10^{16}
(C) 2×10^{16} (D) 1×10^{16}

Sol. The energy emitted by a laser of power $P = 2$ mW in time $t = 1$ s is

$$E_{\text{source}} = Pt = (2 \times 10^{-3})(1) = 2 \times 10^{-3} \text{ J.}$$

The energy of a photon of wavelength $\lambda = 500$ nm is

$$\begin{aligned} E = h\nu &= \frac{hc}{\lambda} = \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{500 \times 10^{-9}} \\ &= 3.96 \times 10^{-19} \text{ J.} \end{aligned}$$

The energy emitted by a laser in one second is equal to the total energy of photons emitted by it in one second

i.e., $E_{\text{source}} = nE$. Substitute values to get number of photons emitted per second as

$$n = \frac{E_{\text{source}}}{E} = \frac{2 \times 10^{-3}}{3.96 \times 10^{-19}} \approx 5 \times 10^{15}.$$

Ans. A \square

Q 2. A 4 kW radiation source produces 10^{20} photons per second. The radiation belong to a part of the spectrum called (2010 m)

- (A) X-rays (B) ultraviolet rays
(C) microwaves (D) γ -rays

Sol. The energy emitted by the source in one second (E_{source}) is equal to the total energy of photons emitted by it in one second i.e., $E_{\text{source}} = nE$. Thus, energy of each photon is

$$E = \frac{E_{\text{source}}}{n} = \frac{Pt}{n} = \frac{(4 \times 10^4)(1)}{10^{20}} = 4 \times 10^{-16} \text{ J.}$$

The energy of a photon is related to the wavelength λ of radiation by $E = hc/\lambda$. Thus, wavelength of the radiation is

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{4 \times 10^{-16}} = 5 \times 10^{-10} \text{ m.}$$

The wavelength $\lambda = 5 \times 10^{-10} \text{ m} = 5 \text{ \AA}$ falls in the X-rays range of electromagnetic spectrum.

Note: A convenient unit of photon energy is electron-volt (eV). To convert energy from joule to eV, divide

the energy in joule by the charge of an electron ($e = 1.6 \times 10^{-19}$ C). Thus, photon energy in this problem is

$$E = \frac{E \text{ (in joule)}}{e} = \frac{4 \times 10^{-16}}{1.6 \times 10^{-19}} = 2500 \text{ eV.}$$

The energy of a photon of wavelength λ is easily obtained by using the relation

$$E = \frac{hc}{\lambda} = \frac{12400 \text{ eV} \cdot \text{\AA}}{\lambda \text{ (in \AA)}}.$$

The order of magnitude of photon energy in microwave range is μeV , in visible range is eV, for X-rays is keV and for gamma rays is MeV.

Ans. A \square

Q 3. A source emits X-rays of wavelength 1 nm and another source emits visible light of wavelength 500 nm. The radiated power of each source is 200 W. The ratio of the number density of photons of X-rays to the number density of photons of the visible light is (2020 m)
(A) 1/250 (B) 500 (C) 250 (D) 1/500

Sol. This question was asked in JEE Main. Usually, *number density* is used for *numbers per unit volume*. The use of *number density of photons* in this question creates ambiguity. If radiation source emits parallel rays (a plane wavefront) then number density is same throughout the radiated volume. However, if source is a point (spherical wavefront) or a line (cylindrical

wavefront) then number density depends on the distance from the source. The question is silent on the type of radiated wavefronts of the two sources.

Let us find the number of photons emitted per second by the two sources. The energy emitted per second by each source is $E_{\text{source}} = Pt = (200)(1) = 200 \text{ J}$.

The energy of each X-ray photon is $E_x = hc/\lambda_x$ and the number of photons emitted per second by the X-ray source is

$$n_x = \frac{E_{\text{source}}}{E_x} = \frac{E_{\text{source}}\lambda_x}{hc}.$$

Similarly, the energy of each visible light photon $E_v = hc/\lambda_v$ and the number of photons emitted per second by the visible light source is

$$n_v = \frac{E_{\text{source}}}{E_v} = \frac{E_{\text{source}}\lambda_v}{hc}.$$

The ratio of the number of photons emitted per second by the X-rays source to the number of photons emitted per second by the visible light source is

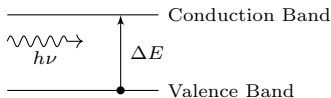
$$\frac{n_x}{n_v} = \frac{\lambda_x}{\lambda_v} = \frac{1}{500}.$$

Note that the number of photons emitted per second by a monochromatic source is directly proportional to its power. Two sources of equal power emit photons flux proportional to their wavelengths.

Ans. D \square

- Q 4.** If a semiconductor photodiode can detect a photon with a maximum wavelength of 400 nm, then its band gap energy is [Given $h = 6.63 \times 10^{-34}$ J·s and $c = 3 \times 10^8$ m/s.] (2020 m)
- (A) 1.5 eV (B) 1.1 eV (C) 3.1 eV (D) 2.0 eV

Sol. In a photodiode, an incident photon gives its energy to a valence band electron. If energy received by the electron is more than the band gap energy then it jumps to the conduction band. Thus, a photodiode can detect a photon only if photon's energy is more than the band gap energy ΔE .



In other words, the band gap energy is equal to the energy of a detected photon of the maximum wavelength i.e.,

$$\Delta E = \frac{hc}{\lambda_{\max}} = \frac{1240 \text{ nm} \cdot \text{eV}}{400 \text{ nm}} = 3.1 \text{ eV}.$$

Ans. C \square

- Q 5.** The power of a point source is 1.0 W. It emits two spectral lines with wavelengths $0.70 \mu\text{m}$ and $0.40 \mu\text{m}$ whose intensities are in a 1:2 ratio, respectively. The photon flux at a distance 1.0 m from the source is

(2020 m)

- (A) $2.4 \times 10^{17} \text{ m}^{-2}\text{s}^{-1}$ (B) $2.0 \times 10^{17} \text{ m}^{-2}\text{s}^{-1}$
 (C) $1.2 \times 10^{17} \text{ m}^{-2}\text{s}^{-1}$ (D) $1.0 \times 10^{17} \text{ m}^{-2}\text{s}^{-1}$

Sol. Let $\lambda_1 = 0.70 \mu\text{m}$ and $\lambda_2 = 0.40 \mu\text{m}$. The photon energy for these wavelengths are $E_1 = hc/\lambda_1$ and $E_2 = hc/\lambda_2$.

Let n_1 and n_2 be photon flux of two spectral lines at a distance r from the point source. The ratio of the intensities of these lines is given by

$$\begin{aligned}\frac{I_1}{I_2} &= \frac{n_1 E_1}{n_2 E_2} \quad \text{i.e.,} \\ \frac{1}{2} &= \frac{n_1 \lambda_2}{n_2 \lambda_1}.\end{aligned}\tag{1}$$

The number of photons emitted per second in these two spectral lines is equal to the number of photons crossing a spherical shell of radius r in one-second i.e., $N_1 = 4\pi r^2 n_1$ and $N_2 = 4\pi r^2 n_2$. The total energy crossing a spherical shell of radius r in one second is equal to the power of the source $P = 1.0 \text{ W}$ i.e.,

$$\begin{aligned}P &= N_1 E_1 + N_2 E_2 \quad \text{i.e.,} \\ 1.0 &= 4\pi r^2 hc \left(\frac{n_1}{\lambda_1} + \frac{n_2}{\lambda_2} \right).\end{aligned}\tag{2}$$

Solve equations (1) and (2) to get

$$\begin{aligned}n_1 &= \frac{\lambda_1}{12\pi r^2 hc}, \\ n_2 &= \frac{\lambda_2}{6\pi r^2 hc}.\end{aligned}$$

Thus, photon flux at a distance $r = 1.0 \text{ m}$ from the

source is

$$\begin{aligned} n &= n_1 + n_2 = \frac{\lambda_1 + 2\lambda_2}{12\pi r^2 hc} \\ &= \frac{0.7 \times 10^{-6} + 2(0.4 \times 10^{-6})}{12 \times 3.14 \times (1.0)^2 \times (6.63 \times 10^{-34})(3 \times 10^8)} \\ &= 2 \times 10^{17} \text{ m}^{-2} \text{ s}^{-1}. \end{aligned}$$

Ans. B \square

Radiations Momentum

Q 6. Photon of frequency ν has a momentum associated with it. If c is the velocity of light, the momentum is (2007 m)

(A) $h\nu/c$ (B) ν/c (C) $h\nu c$ (D) $h\nu/c^2$

Sol. The wavelength of a photon of momentum p is given by $\lambda = h/p$. The wavelength is related to frequency by $c = \nu\lambda$. Thus, momentum of a photon of frequency ν is

$$p = \frac{h}{\lambda} = \frac{h}{c/\nu} = \frac{h\nu}{c}.$$

Ans. A \square

Q 7. Light is incident normally on a completely absorbing surface with an energy flux of 25 W/cm^2 . If the surface has an area of 25 cm^2 , the momentum transferred to the surface in 40 min time duration will be (2019 m)

(A) $6.3 \times 10^{-4} \text{ N-s}$ (B) $1.4 \times 10^{-6} \text{ N-s}$
(C) $5.0 \times 10^{-3} \text{ N-s}$ (D) $3.5 \times 10^{-6} \text{ N-s}$

Sol. The momentum flux (i.e., momentum per unit area per unit time) of a radiation of energy flux S is given by S/c , where c is the speed of light. Since radiation is completely absorbed by the surface, its total momentum is transferred to the surface. Thus, momentum transferred per second to a completely absorbing surface of area $A = 25 \text{ cm}^2$ by a radiation of energy flux $S = 25 \text{ W/cm}^2$ is $(S/c)A$. The momentum transferred to the surface in time $t = 40 \text{ min}$ is

$$p = \left(\frac{S}{c}\right) At = \frac{(25)(25)(40 \times 60)}{3 \times 10^8} = 5 \times 10^{-3} \text{ N}\cdot\text{s}.$$

Aliter: Let us see radiation as a stream of photons. Let ν be the radiation frequency. The energy and momentum of each photon are $h\nu$ and $h\nu/c$, respectively. The total energy absorbed by the surface of area A in time t is $E = SAt$. Thus, the total number of photons absorbed by the surface are

$$n = \frac{E}{h\nu} = \frac{SAt}{h\nu}.$$

The momentum transferred to the surface by n photons, each of momentum $h\nu/c$, is

$$p = n \left(\frac{h\nu}{c}\right) = \frac{SAt}{h\nu} \frac{h\nu}{c} = \frac{SAt}{c}.$$

Ans. C \square

Q 8. Sunlight of energy density 50 W/m^2 is normally incident on the surface of a solar panel. Some part of

incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1 m^2 surface area will be close to [Given $c = 3 \times 10^8 \text{ m/s}$] (2019 m)

- (A) $15 \times 10^{-8} \text{ N}$ (B) $20 \times 10^{-8} \text{ N}$
 (C) $10 \times 10^{-8} \text{ N}$ (D) $35 \times 10^{-8} \text{ N}$

Sol. Instead of *energy density*, the question should have used the term *energy flux* $S = 50 \text{ W/m}^2$ (energy crossing per unit area per unit time normal to the surface). The energy falling per second on a solar panel of area $A = 1 \text{ m}^2$ in one-second is

$$E = SA = (50)(1) = 50 \text{ J.}$$

Out of the energy E , 75% is absorbed ($E_a = 0.75E$) and 25% is reflected ($E_r = 0.25E$). The momentum transferred per second to the surface (i.e., force exerted) by the absorbed radiation is $F_a = E_a/c$.

The reflected radiation carry the same momentum as the incident radiation, but in the opposite direction. Thus, the momentum transferred per second by the reflected radiation is $F_r = 2(E_r/c)$. It is similar to the momentum transferred in an elastic collision when a light ball of mass m strikes a stationary heavy ball of mass $M (\gg m)$.

Thus, force exerted on the solar panel is

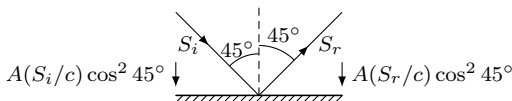
$$\begin{aligned} F &= F_a + F_r = \frac{E_a}{c} + \frac{2E_r}{c} = (0.75 + 2 \times 0.25) \frac{SA}{c} \\ &= \frac{(1.25)(50)(1)}{3 \times 10^8} = 20.8 \times 10^{-8} \text{ N.} \end{aligned}$$

- Q 9.** A light wave of intensity 0.2 W/cm^2 falls on a plane mirror surface of reflection coefficient 0.8. The angle of incidence is 45° . The pressure exerted by the radiation on the mirror is
 (A) $6 \times 10^{-6} \text{ N/m}^2$ (B) $8.5 \times 10^{-6} \text{ N/m}^2$
 (C) $8.5 \times 10^{-8} \text{ N/m}^2$ (D) $6 \times 10^{-8} \text{ N/m}^2$

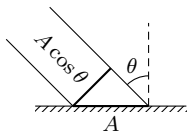
Sol. The intensity (or energy flux) of the incident light is $S_i = 0.2 \text{ W/cm}^2 = 2000 \text{ W/m}^2$ and that of the reflected light is $S_r = 0.8S_i = 1600 \text{ W/m}^2$.

The radiation pressure on the mirror is equal to the *normal* force exerted by the radiation on unit area. The tangential force exerted by the radiation will not contribute to it.

The momentum flux of incident radiation is S_i/c and that of the reflected radiation is S_r/c . Both makes an angle $\theta = 45^\circ$ with the mirror surface.



Consider an area A of the mirror. The radiations lying in a cross-sectional area $A \cos \theta$ will fall on the area A . The momentum transferred per second to the mirror by incident radiation is $A \cos \theta (S_i/c)$. The normal component of this momentum is $A(S_i/c) \cos^2 \theta$. Its direction is into the mirror.



The momentum carried by the radiations emanating from the area A is $A \cos \theta (S_r/c)$. The normal component of this momentum is $A(S_r/c) \cos^2 \theta$. By momentum conservation, the momentum transferred per second to the mirror is $A(S_r/c) \cos^2 \theta$ and its direction is into the mirror.

Thus, normal component of the momentum transferred per second to $A = 1 \text{ m}^2$ area of the mirror (i.e., radiation pressure) is

$$\begin{aligned} p &= A(S_i/c) \cos^2 45^\circ + A(S_r/c) \cos^2 45^\circ \\ &= (1 + 0.8) \frac{(1)(2 \times 10^3)(0.5)}{3 \times 10^8} \\ &= 6 \times 10^{-6} \text{ N/m}^2. \end{aligned}$$

We encourage you to solve this problem by considering light as a stream of photons.

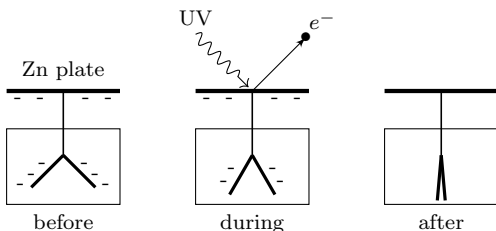
Ans. A \square

Photoelectric Effect: Conceptual Questions

Q 10. An early study on photoelectric effect was carried out by a German physicist W. Hallwachs. He studied the effect of UV light on a charged electroscope. Which of the following option(s) is (are) not correct?

- (A) The leaves of a negatively charged electroscope converge when it is illuminated with UV radiations.
- (B) The leaves of a positively charged electroscope don't show noticeable effect when it is illuminated with UV radiations.
- (C) The leaves of a neutral electroscope don't show noticeable effect when it is illuminated with UV radiations.
- (D) UV radiations can eject electrons from the surface of all metals.

Sol. The electroscope used by W. Hallwachs was fitted with a zinc plate as shown in the figure. The work function of Zn is 4.3 eV. The ultraviolet radiations of wavelength less than 290 nm can emit electrons from the zinc surface.



Initially, the electroscope was negatively charged. Its leaves are diverged. Then, zinc plate is illuminated by UV radiations. These radiations eject photoelectrons from the zinc surface. The electroscope gets discharged. The leaves converge.

In a neutral or positively charged electroscope, the leaves of an electroscope don't show appreciable effect when it is illuminated by UV radiations. The ejection of photoelectrons from the zinc plate make the electroscope positively charged. These positive charges attracts the ejected photoelectrons and the process stops.

To eject a photoelectron, the energy of incident photon shall be more than the work function of the metal. Hence, UV radiations can eject photoelectrons only from metals having work function less than the energy of UV photon e.g., from metals like sodium, aluminium, zinc etc.

Ans. D \square

Q 11. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV falls on it is 4 eV. The stopping potential is (1997)
(A) 2 V (B) 4 V (C) 6 V (D) 10 V

Sol. In photoelectric effect, stopping potential is defined as the potential required to stop the photoelectrons with maximum kinetic energy. Thus, the stopping potential is 4 V. Note that the work function in this case is $\phi = 2$ eV.

Ans. B \square

Q 12. The dimension of stopping potential V_0 in photoelectric effect in units of Planck's constant h , speed of light c , Gravitational constant G , and ampere A is

- (2020 m)
(A) $h^{-2/3}c^{-1/3}G^{4/3}A^{-1}$ (B) $h^{2/3}c^{5/3}G^{1/3}A^{-1}$
(C) $h^2G^{3/2}c^{1/3}A^{-1}$ (D) $h^0c^5G^{-1}A^{-1}$

Sol. The dimensions of given parameters are: stopping potential $[V_0] = [ML^2T^{-3}A^{-1}]$, Planck's constant $[h] = [ML^2T^{-1}]$, speed of light $[c] = [LT^{-1}]$ and Gravitational constant $[G] = [M^{-1}L^3T^{-2}]$.

The dimensions of stopping potential can be written as $[V_0] = [h]^x [c]^y [G]^z [A]^w$ for some constants x, y, z and w . Substitute dimensions of various parameters to get

$$ML^2T^{-3}A^{-1} = [ML^2T^{-1}]^x [LT^{-1}]^y \times [M^{-1}L^3T^{-2}]^z [A]^w.$$

Use principle of dimensional homogeneity to get $x = 0$, $y = 5$, $z = -1$ and $w = -1$. Thus, dimensions of stopping potential in terms of given parameters are $V_0 = [h^0 c^5 G^{-1} A^{-1}]$.

Ans. D \square

Q 13. Photoelectric effect supports quantum nature of light because (1987)

- (A) there is a minimum frequency of light below which no photoelectrons are emitted.
- (B) the maximum kinetic energy of photoelectrons depends only on the frequency of light and not on intensity.
- (C) even when the metal surface is faintly illuminated, the photoelectrons leave the surface immediately.
- (D) electric charge of the photoelectrons is quantized.

Sol. According to Einstein's quantum theory of the photoelectric effect, light constitutes of photons. The

energy of each photon is $E = h\nu$, where ν is frequency of light. The photoelectrons are ejected when a photon of energy greater than work function (ϕ) collides with an electron on metal surface. Maximum kinetic energy of ejected photoelectrons, $K_{\max} = h\nu - \phi$, depends on the frequency of incident radiation and not on intensity. The photoelectrons are ejected if photon's energy is greater than the work function, irrespective of the intensity of light. The numbers of photons striking the metal surface per second and the number of photoelectrons ejected per second depend on the intensity of incident radiation.

Ans. A, B, C \square

Q 14. The threshold wavelength for photoelectric emission from a material is 5200 \AA . Photoelectrons will be emitted when this material is illuminated with monochromatic radiations from a (1982)

- (A) 50 W infrared lamp
- (B) 1 W infrared lamp
- (C) 50 W ultraviolet lamp
- (D) 1 W ultraviolet lamp

Sol. The emission of the photoelectrons from a metal surface depends on the frequency (wavelength) of incident radiation and not on its intensity. The wavelength of incident radiation should be less than or equal to the threshold wavelength of 5200 \AA . The wavelength of ultraviolet radiation is less than 5200 \AA . Hence, the ultraviolet lamp of 50 W as well as 1 W will emit photoelectrons.

Ans. C, D \square

Q 15. The time taken by a photoelectron to come out after the photon strikes is approximately (2006 m)
(A) 10^{-1} s (B) 10^{-4} s (C) 10^{-10} s (D) 10^{-16} s

Sol. The photoelectric effect is an instantaneous phenomenon. The time delay between incidence of photon and emission of electron is of the order of 10^{-10} s. This observation could not be explained by the wave theory. It supports the quantum nature of the light.

Ans. C \square

Q 16. The photoelectric process requires a bound electron. If a free electron absorb a photon then
(A) linear momentum is not conserved.
(B) angular momentum is not conserved.
(C) energy is not conserved.
(D) the energy and linear momentum are not conserved simultaneously.

Sol. In photoelectric effect, an electron absorb a photon and comes out of the metal surface.

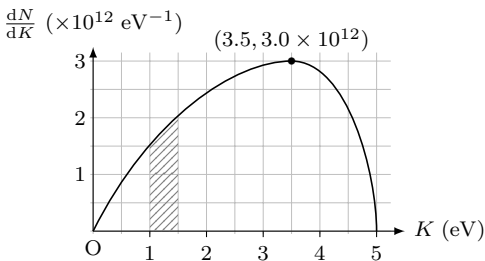
In solid-state physics, the 'free electron model' is used to explain electrical and thermal properties of metals. In this model, the valence electrons are free to move in the spaces between ions. These 'free electrons' are responsible for high electrical and thermal conductivities of metals. However, these 'free electrons' are not 'free' in real sense. They are bound by the metal. An

energy (equal to the work function) is required to do work against the binding forces of the metal.

A free electron (i.e., an isolated electron) cannot absorb a photon as it is not possible to satisfy the energy and linear momentum conservation simultaneously. This can be proved using a few formulas of special theory of relativity.

Ans. D \square

Q 17. The kinetic energy of an ejected photoelectron varies from zero to a maximum value. The distribution function $f = dN/dK$ indicates photoelectrons lying in various energy ranges. The number of photoelectrons having the kinetic energy from K to $K + dK$ is given by $f dK$. The distribution function for a photoelectric effect experiment is given in the figure. Select the correct option(s)?



- (A) the stopping potential is 5 V.
- (B) the stopping potential is 3.5 V.
- (C) the saturation photocurrent is $\approx 1.6 \mu\text{A}$.
- (D) the saturation photocurrent is $\approx 0.48 \mu\text{A}$.

Sol. The maximum kinetic energy of photoelectrons is $K_{\max} = 5 \text{ eV}$. Thus, stopping potential is $V_0 = K_{\max}/e = 5 \text{ V}$.

The saturation current flows when all the ejected photoelectrons reach the collector. The total number of photoelectrons ejected per second is given by the area under the curve. You can count the number of squares to get the area under the curve. There are approximately 40 squares each of area 0.25×10^{12} . Thus, total number of photoelectrons ejected per second is

$$N = \int_0^5 f dK \approx 40(0.25 \times 10^{12}) = 10^{13}.$$

The charge flowing per second i.e., saturation current is

$$i_s = eN = (1.6 \times 10^{-19})(10^{13}) = 1.6 \mu\text{A}.$$

Ans. A, C \square

Q 18. The classical electron radius is obtained by comparing its electrostatic energy and rest mass energy. It is given by

$$r = \frac{1}{4\pi\epsilon_0} \frac{e^2}{m_e c^2} = 2.82 \times 10^{-15} \text{ m}.$$

Let light of intensity 500 W/m^2 falls normally on a disc of radius r . Assume light is classical wave and it is completely absorbed. The time required to accumulate 1.0 eV of energy is approximately

(A) 10^{-9} s (B) 10^{-7} s (C) 10^3 s (D) 10^7 s

Sol. The area of the disc is

$$A = \pi r^2 = 3.14 (2.82 \times 10^{-15})^2 = 2.5 \times 10^{-29} \text{ m}^2.$$

The energy absorbed by the disc per second is

$$P = IA = (500)(2.5 \times 10^{-29}) = 1.25 \times 10^{-26} \text{ J/s.}$$

Thus, time required to accumulate energy $E = 1.0 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ is

$$t = \frac{E}{P} = \frac{1.6 \times 10^{-19}}{1.25 \times 10^{-26}} = 1.28 \times 10^7 \text{ s} = 148 \text{ days.}$$

In photoelectric effect, the intensity of incident light and the energy of photoelectron are of the same order as given in this problem. In the classical picture (electron as sphere and light as wave), the time required for an electron to get sufficient energy for its ejection is huge. This is not in agreement with the experimental observations. In the quantum picture of light, a photon (or energy quanta) interacts with an electron and gives its energy to the electron. The electron is ejected almost instantaneously.

Ans. B \square

Einstein's Equation

Q 19. In a photoelectric emission process the maximum energy of the photoelectrons increases with increasing intensity of the incident light. (true/false) (1986)

Sol. The maximum kinetic energy of the photoelectron ejected from the metal of work function ϕ is given by, $K_{\max} = h\nu - \phi$, where ν is the frequency of the incident radiation. The maximum kinetic energy K_{\max} depends on the frequency of the incident radiation and not on its intensity. It is an important argument in support of quantum nature of light.

Ans. F \square

Q 20. The kinetic energy of photoelectrons emitted by a photosensitive surface depends on the (1981)

- (A) intensity of the incident radiation.
- (B) frequency of the incident radiation.
- (C) stopping potential
- (D) work function

Sol. An incident photon of energy $h\nu$ gives its energy to an electron. After receiving this energy, the electron may lose some (or all) of its energy due to collision with atoms ($\phi_{\text{collision}}$). Also, an electron that comes out of the surface loses a part of its energy in doing work against the binding forces (ϕ). Thus, kinetic energy of an ejected photoelectron can be written as

$$K = h\nu - \phi - \phi_{\text{collision}}.$$

The kinetic energy of the ejected photoelectrons depend on the frequency ν as well as on the work function ϕ .

If an electron don't lose its energy due to collision (i.e., $\phi_{\text{collision}} = 0$) then it is ejected with the maximum

kinetic energy. The maximum kinetic energy of photoelectrons emitted by a photosensitive surface is given by $K_{\max} = h\nu - \phi$.

Ans. B, D \square

Q 21. The maximum kinetic energy of electron emitted in the photoelectric effect is linearly dependent on the of the incident radiation. (1984)

Sol. The maximum kinetic energy of the ejected photoelectrons is linearly dependent on the frequency of the incident radiation i.e., $K_{\max} = h\nu - \phi$. Here, linearity shall not be confused with proportionality. Doubling the frequency will not double the maximum kinetic energy.

Ans. Frequency \square

Q 22. The work function of a substance is 4 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately (1998)
(A) 540 nm (B) 400 nm (C) 310 nm (D) 220 nm

Sol. In photoelectric effect, the wavelength λ of incident photon, work function ϕ of the material, and the maximum kinetic energy K_{\max} of the ejected photoelectron are related by

$$hc/\lambda = \phi + K_{\max}.$$

The longest wavelength corresponds to $K_{\max} = 0$. Substitute the values in the above equation to get

$$\lambda_{\max} = \frac{hc}{\phi} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{4 \times 1.6 \times 10^{-19}} = 310 \text{ nm.}$$

Ans. C \square

Q 23. The work functions of sodium and copper are 2.3 eV and 4.5 eV, respectively. The ratio of the cut-off wavelengths of the sodium to that of the copper is close to

(2002 m)

(A) 1 : 2 (B) 4 : 1 (C) 2 : 1 (D) 1 : 4

Sol. The cut-off wavelength (i.e., the maximum wavelength of radiation that can eject photoelectrons) for a metal of work function ϕ is given by $\lambda_c = hc/\phi$. Thus, the ratio of the cut-off wavelengths of the sodium to that of the copper is

$$\frac{\lambda_{c, \text{Na}}}{\lambda_{c, \text{Cu}}} = \frac{hc/\phi_{\text{Na}}}{hc/\phi_{\text{Cu}}} = \frac{\phi_{\text{Cu}}}{\phi_{\text{Na}}} = \frac{4.5}{2.3} \approx 2.$$

We encourage you to calculate cut-off wavelength for sodium. Can visible light eject photoelectrons from the sodium surface?

Ans. C \square

Q 24. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV, and the stopping potential for a radiation incident on this surface of 5 V. The incident radiation lies in

(2006 m)

(A) X-ray region (B) ultraviolet region
(C) infrared region (D) visible region

Sol. The work function of the metal is equal to the energy of a photon of threshold frequency ν_0 i.e., $\phi = h\nu_0 = 6.2$ eV. The maximum kinetic energy of the photoelectron is equal to the change in its potential energy when subjected to a potential difference V_0 i.e., $K_{\max} = eV_0 = 5$ eV. By Einstein's equation, $K_{\max} = h\nu - \phi$, the energy of a photon of incident radiation is

$$h\nu = K_{\max} + \phi = 5 + 6.2 = 11.2 \text{ eV.}$$

The photon of energy $E = 11.2$ eV (or wavelength $\lambda = hc/E = 12400/11.2 = 1107 \text{ \AA}$) lies in UV region of electromagnetic spectrum.

Exercise: Electrons in a photoelectric effect experiment emerge from an aluminium surface ($\phi = 4.2$ eV) with a maximum kinetic energy of 1.3 eV. What is the wavelength of the light? *Ans.* 2255 \AA .

Ans. B \square

Maximum KE of Photoelectrons

Q 25. Light of wavelength λ eject photoelectrons from a metal surface with maximum kinetic energy K . When wavelength is changed to λ' , the maximum kinetic energy of photoelectrons increased to $2K$. Which of the following relation is correct?

- (A) $\lambda' < \lambda/2$ (B) $\lambda' > \lambda/2$
(C) $\lambda < \lambda' < \lambda/2$ (D) $\lambda' = \lambda/2$

Sol. Let ϕ be work function of the metal. Substitute given values in Einstein's equation, $K_{\max} = hc/\lambda - \phi$,

to get

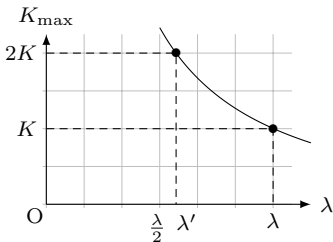
$$K = hc/\lambda - \phi, \quad (1)$$

$$2K = hc/\lambda' - \phi. \quad (2)$$

Eliminate ϕ from equations (1) and (2) to get

$$\lambda' = \frac{\lambda}{1 + \frac{K}{hc/\lambda}}. \quad (3)$$

Note that $\frac{K}{hc/\lambda} > 0$ and $\frac{K}{hc/\lambda} < 1$ from equation (1). Use these inequalities in equation (3) to get $\lambda' < \lambda$ and $\lambda' > \lambda/2$.



Note that decreasing the wavelength by half will increase the maximum kinetic energy by more than twice.

Ans. C \square

Q 26. When the wavelength of radiation falling on a metal is changed from 500 nm to 200 nm, the maximum kinetic energy of the photoelectrons becomes three

times larger. The work function of the metal is close to
(2020 m)

(A) 0.52 eV (B) 1.02 eV (C) 0.81 eV (D) 0.61 eV

Sol. Substitute values of the maximum kinetic energies and wavelengths in equation $K_{\max} = hc/\lambda - \phi$ to get

$$K = \frac{hc}{\lambda_1} - \phi = \frac{1240 \text{ nm} \cdot \text{eV}}{500 \text{ nm}} - \phi = 2.48 - \phi, \quad (1)$$

$$3K = \frac{hc}{\lambda_2} - \phi = \frac{1240 \text{ nm} \cdot \text{eV}}{200 \text{ nm}} - \phi = 6.20 - \phi. \quad (2)$$

Divide equation (2) by (1) and then solve for ϕ to get the work function $\phi = 0.62 \text{ eV}$.

Ans. D \square

Q 27. In a photoelectric effect experiment the threshold wavelength of light is 380 nm. If the wavelength of incident light is 260 nm, the maximum kinetic energy of emitted electrons will be
(2019 m)

(A) 1.5 eV (B) 3.0 eV (C) 4.5 eV (D) 15.1 eV

Sol. The work function of a material is equal to the energy of a photon of threshold (or cut-off) wavelength $\lambda_c = 380 \text{ nm}$ i.e., $\phi = hc/\lambda_c$. The maximum kinetic energy of emitted electron when illuminated with light

of wavelength $\lambda = 260 \text{ nm}$ is given by

$$\begin{aligned} K_{\max} &= \frac{hc}{\lambda} - \phi = \frac{hc}{\lambda} - \frac{hc}{\lambda_c} \\ &= \frac{1240 \text{ nm} \cdot \text{eV}}{260 \text{ nm}} - \frac{1240 \text{ nm} \cdot \text{eV}}{380 \text{ nm}} = 1.51 \text{ eV}. \end{aligned}$$

Ans. A \square

Q 28. The magnetic field associated with a light wave is given, at the origin, by

$$B = B_0 [\sin(3.14 \times 10^7 ct) + \sin(6.28 \times 10^7 ct)].$$

If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photoelectrons? [Given $c = 3 \times 10^8 \text{ m/s}$ and $h = 6.6 \times 10^{-34} \text{ J} \cdot \text{s}$] (2019 m)

(A) 6.82 eV (B) 12.5 eV (C) 8.52 eV (D) 7.72 eV

Sol. The magnetic field at the origin

$$B = B_0 [\sin(3.14 \times 10^7 ct) + \sin(6.28 \times 10^7 ct)],$$

is the superposition of two fields

$$B = B_1 + B_2 = B_0 \sin(2\pi\nu_1 t) + B_0 \sin(2\pi\nu_2 t),$$

of frequencies

$$\begin{aligned} \nu_1 &= (0.5 \times 10^7)(3 \times 10^8) = 1.5 \times 10^{15} \text{ Hz}, \quad \text{and} \\ \nu_2 &= (10^7)(3 \times 10^8) = 3 \times 10^{15} \text{ Hz}. \end{aligned}$$

Thus, the incident radiation consists of two frequencies: $\nu_1 = 1.5 \times 10^{15}$ Hz and $\nu_2 = 3 \times 10^{15}$ Hz. The energy of photons of these radiations are

$$\begin{aligned} E_1 &= h\nu_1 = (6.6 \times 10^{-34})(1.5 \times 10^{15}) \text{ J} \\ &= \frac{(6.6 \times 10^{-34})(1.5 \times 10^{15})}{1.6 \times 10^{-19}} = 6.2 \text{ eV}, \\ E_2 &= h\nu_2 = 12.4 \text{ eV}. \end{aligned}$$

The energy of photons of each radiation is more than the work function $\phi = 4.7$ eV. Hence, both radiations will eject photoelectrons from the silver plate. The maximum kinetic energy of photoelectrons ejected by these radiations are

$$\begin{aligned} K_{\max,1} &= E_1 - \phi = 6.2 - 4.7 = 1.5 \text{ eV}, \\ K_{\max,2} &= E_2 - \phi = 12.4 - 4.7 = 7.7 \text{ eV}. \end{aligned}$$

The maximum kinetic energy of all photoelectrons emitted from the silver plate is the maximum of $K_{\max,1}$ and $K_{\max,2}$ i.e., $K_{\max} = K_{\max,2} = 7.7$ eV.

Note that the stopping potential depends only on the higher frequency ν_2 . However, the contribution of each frequency to the saturation current is equal. Show that the saturation current will become half when ν_1 is reduced from 1.5×10^{15} Hz to 10^{15} Hz.

Ans. D \square

Q 29. The surface of a metal is illuminated with the light of 400 nm. The maximum kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The

work function of the metal is [Given $hc = 1240 \text{ eV} \cdot \text{nm}$]
(2009 m)

(A) 3.09 eV (B) 1.41 eV (C) 1.51 eV (D) 1.68 eV

Sol. Substitute the given values in Einstein's equation $K_{\max} = hc/\lambda - \phi$ to get the work function of the metal

$$\phi = \frac{hc}{\lambda} - K_{\max} = \frac{1240 \text{ eV} \cdot \text{nm}}{400 \text{ nm}} - 1.68 = 1.42 \text{ eV}.$$

Ans. B \square

Q 30. In a photoelectric experiment, the collector plate is at a positive potential of 2.0 V with respect to the emitter plate made up of copper ($\phi = 4.5 \text{ eV}$). The emitter is illuminated by a monochromatic light of wavelength 200 nm. Let K_{\min} and K_{\max} be the minimum and the maximum kinetic energies of the photoelectrons at the emitter and K_{\min}^c and K_{\max}^c be the minimum and the maximum kinetic energies of photoelectrons at the collector. Which of the following option(s) is (are) correct?

- (A) $K_{\min} = 0$ (B) $K_{\max} = 1.7 \text{ eV}$
(C) $K_{\min}^c = 2 \text{ eV}$ (D) $K_{\max}^c = 3.7 \text{ eV}$

Sol. The energy of a photon of wavelength 200 nm is

$$E = hc/\lambda = 1240/200 = 6.2 \text{ eV}.$$

The energy received by an electron from a photon is 6.2 eV. The electron may lose its entire energy to the

metal due to collision with atoms. Thus, the minimum kinetic energy of the photoelectron at the emitter is

$$K_{\min} = 0.$$

An electron may not lose any energy if it skips collision. However, to come out of the metal, it need to do work against the binding forces of the metal. This work is equal to the work function $\phi = 4.5$ eV. Thus, the maximum kinetic energy of the photoelectrons (at the emitter) is

$$K_{\max} = hc/\lambda - \phi = 6.2 - 4.5 = 1.7 \text{ eV}.$$

The collector plate is at a positive potential of $V = 2$ V with respect to the emitter. Thus, each photoelectron gains a potential energy $U = eV = 2$ eV while travelling from the emitter to the collector. Hence, the minimum and the maximum energy of photoelectrons at the collector are

$$K_{\min}^c = K_{\min} + U = 0 + 2 = 2 \text{ eV},$$

$$K_{\max}^c = K_{\max} + U = 1.7 + 2 = 3.7 \text{ eV}.$$

Ans. A, B, C, D \square

Q 31. The maximum kinetic energy of photoelectrons is 2.8 eV. When the wavelength of the light is increased by 50%, the maximum kinetic energy decreases to 1.1 eV. Which of the following is (are) correct?

- (A) The work function is $\phi = 2.3$ eV.
- (B) The work function is $\phi = 4.6$ eV.
- (C) The initial wavelength of light is 244 nm.
- (D) The final wavelength of light is 344 nm.

Sol. Substitute given values in $K_{\max} = hc/\lambda - \phi$ to get

$$2.8 = hc/\lambda - \phi, \quad (1)$$

$$1.1 = hc/1.5\lambda - \phi. \quad (2)$$

Solve equations (1) and (2) to get the initial wavelength of light $\lambda = 244$ nm and the work function $\phi = 2.3$ eV.

Ans. A, C \square

Maximum Velocity of Photoelectrons

Q 32. Two identical photocathodes receive light of frequencies ν_1 and ν_2 . If the velocities of the photoelectrons (of mass m) coming out are respectively v_1 and v_2 , then (2003 m)

(A) $v_1^2 - v_2^2 = \frac{2h}{m}(\nu_1 - \nu_2)$

(B) $v_1 + v_2 = \left[\frac{2h}{m}(\nu_1 + \nu_2)\right]^{1/2}$

(C) $v_1^2 + v_2^2 = \frac{2h}{m}(\nu_1 + \nu_2)$

(D) $v_1 - v_2 = \left[\frac{2h}{m}(\nu_1 - \nu_2)\right]^{1/2}$

Sol. Note that v_1 and v_2 are the *maximum* velocities of the photoelectrons for the incident frequencies ν_1 and ν_2 , respectively. The qualifier *maximum* should have been mentioned in the question. Substitute given parameters in equation $K_{\max} = h\nu - \phi$ to get

$$\frac{1}{2}mv_1^2 = h\nu_1 - \phi, \quad (1)$$

$$\frac{1}{2}mv_2^2 = h\nu_2 - \phi. \quad (2)$$

Subtract equation (2) from (1) and simplify to get

$$v_1^2 - v_2^2 = \frac{2h}{m}(\nu_1 - \nu_2).$$

Ans. A \square

Q 33. Radiation of wavelength λ , is incident on a photocell. The fastest emitted electron has speed v . If the wavelength is changed to $3\lambda/4$, the speed v' of the fastest emitted electron will be (2016 m)

- (A) $v' > v(4/3)^{\frac{1}{2}}$ (B) $v' < v(4/3)^{\frac{1}{2}}$
 (C) $v' = v(4/3)^{\frac{1}{2}}$ (D) $v' = v(3/4)^{\frac{1}{2}}$

Sol. The fastest emitted electron will have the maximum kinetic energy. Thus, $K_{\max} = mv^2/2$ for the wavelength λ and $K_{\max} = mv'^2/2$ for the wavelength $3\lambda/4$. Substitute these values in equation $K_{\max} = hc/\lambda - \phi$ to get

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi, \quad (1)$$

$$\frac{1}{2}mv'^2 = \frac{hc}{(3\lambda/4)} - \phi. \quad (2)$$

Subtract equation (1) from (2) and simplify to get

$$v' = \sqrt{v^2 + \frac{2hc}{3\lambda m}}. \quad (3)$$

From equation (1),

$$\frac{2hc}{\lambda m} = v^2 + \frac{2\phi}{m} > v^2. \quad (4)$$

Use inequality (4) in equation (3) to get

$$v' = \sqrt{v^2 + \frac{2hc}{3\lambda m}} > \sqrt{v^2 + \frac{v^2}{3}} = \sqrt{\frac{4}{3}} v.$$

Ans. A \square

Q 34. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are v_1 and v_2 , respectively. If the ratio $v_1 : v_2 = 2 : 1$ and $hc = 1240 \text{ eV} \cdot \text{nm}$, the work function of the metal is nearly (2014)

(A) 3.7 eV (B) 3.2 eV (C) 2.8 eV (D) 2.5 eV

Sol. The maximum kinetic energy of ejected photoelectrons is given by

$$K_{\max} = \frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi. \quad (1)$$

Substitute the values in equation (1) to get

$$\frac{1}{2}mv_1^2 = \frac{1240}{248} - \phi, \quad (2)$$

$$\frac{1}{2}mv_2^2 = \frac{1240}{310} - \phi. \quad (3)$$

Divide equation (2) by (3) and use $v_1/v_2 = 2$ to get $\phi = 3.7 \text{ eV}$.

Ans. A \square

Q 35. Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350$ nm and then, by light of wavelength $\lambda_2 = 540$ nm. It is found that the maximum speed of the photoelectrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to (2019 m)

(A) 1.8 (B) 2.5 (C) 5.6 (D) 1.4

Sol. The photoelectrons of the maximum kinetic energy (K_{\max}) have the maximum speed (v_{\max}) given by $K_{\max} = \frac{1}{2}mv_{\max}^2$. The maximum speed of the photoelectrons decreases when wavelength of the incident light is increased. Let v_1 and v_2 be the maximum speeds of the photoelectrons for wavelengths $\lambda_1 = 350$ nm and $\lambda_2 = 540$ nm, respectively. Substitute these values in equation $K_{\max} = hc/\lambda - \phi$ to get

$$\frac{1}{2}mv_1^2 = \frac{hc}{\lambda_1} - \phi = \frac{1240}{350} - \phi, \quad (1)$$

$$\frac{1}{2}mv_2^2 = \frac{hc}{\lambda_2} - \phi = \frac{1240}{540} - \phi, \quad (2)$$

$$v_2 = v_1/2. \quad (3)$$

Solve equations (1)–(3) to get $\phi = 1.8$ eV.

Exercise: The surface of a metal is illuminated alternately with photons of energies $E_1 = 4$ eV and $E_2 = 2.5$ eV respectively. The ratio of maximum speeds of the photoelectrons emitted in the two cases is 2. The work function of the metal is? *Ans.* 4.5 eV.

Ans. A \square

Q 36. The relativistic expression for electron's kinetic energy is recommended if the speed of photoelectron is greater than 10% of the speed of light. For photoelectrons ejected from an aluminium surface ($\phi = 4.2$ eV) what is the smallest wavelength of an incident photon for which classical expression may be used?
 (A) 4800 Å (B) 480 Å (C) 48 Å (D) 4.8 Å

Sol. We need to find the wavelength λ of an incident photon that eject electron from aluminium with the maximum speed $v = 0.1c = 0.1 \times 3 \times 10^8 = 3 \times 10^7$ m/s. The kinetic energy of the electron is

$$\begin{aligned} K_{\max} &= \frac{1}{2}mv^2 = \frac{1}{2}(9.1 \times 10^{-31})(0.1 \times 3 \times 10^8)^2 \text{ J} \\ &= \frac{\frac{1}{2}(9.1 \times 10^{-31})(0.1 \times 3 \times 10^8)^2}{1.6 \times 10^{-19}} \text{ eV} \\ &= 2560 \text{ eV.} \end{aligned}$$

Use equation $K_{\max} = hc/\lambda - \phi$ to get the wavelength

$$\lambda = \frac{hc}{K_{\max} + \phi} = \frac{12400 \text{ Å} \cdot \text{eV}}{2560 + 4.2} = 4.8 \text{ Å.}$$

This wavelength lies in X-rays region of the electromagnetic spectrum. Thus, non-relativistic expression for electron's kinetic energy may give erroneous results for high energy photons. Note that the photon energy is much larger than the work function.

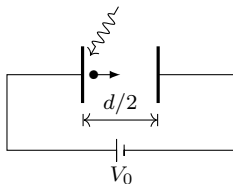
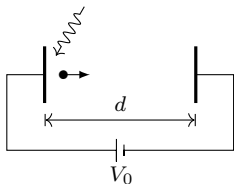
Ans. D \square

Stopping Potential

Q 37. In a photoelectric effect experiment, the potential difference between the emitter and the collector is such that the fastest photoelectrons can fly only one-half of the distance between the emitter and the collector. If the distance between the emitter and the collector is reduced by half without changing the applied voltage then the fastest photoelectrons will

- (A) reach only the midpoint i.e., they fly only one-half of the distance between the emitter and the collector.
- (B) fly only one-fourth of the distance between the emitter and the collector.
- (C) fly only three-fourth of the distance between the emitter and the collector.
- (D) reach the collector.

Sol. For simplicity, let us assume the emitter (cathode) and the collector (anode) as two parallel plates of a capacitor. Also, assume that photoelectrons travel in a direction perpendicular to the plates. The collector is at a negative potential V_0 relative to the emitter.



In the first case, the plates are separated by a distance d . The electric field between the plates is $E = V_0/d$. The repelling force on the electron is $F = eE = eV_0/d$. The fastest photoelectron loses its kinetic energy in travelling half the distance i.e.,

$$K_{\max} = (eE)(d/2) = eV_0/2. \quad (1)$$

In the second case, the distance between the plates is reduced to $d/2$. The electric field between the plates increases to $E = V_0/(d/2) = 2V_0/d$. The repelling force on the electron is $F = eE = 2eV_0/d$. The fastest photoelectron loses its kinetic energy in travelling a distance x given by

$$K_{\max} = Fx = 2eV_0x/d. \quad (2)$$

Eliminate K_{\max} from equations (1) and (2) to get $x = d/4$. Thus, photoelectron can fly only half of the distance between the plates. Note that the stopping potential is independent of the distance between the emitter and the collector.

Ans. A \square

Q 38. The electric field of a light wave is given as

$$\vec{E} = 10^{-3} \cos \left(\frac{2\pi x}{5 \times 10^{-7}} - 2\pi \times 6 \times 10^{14} t \right) \hat{i} \text{ N/C.}$$

This light falls on a metal plate of work function 2 eV. The stopping potential of the photoelectrons is

(2019 m)

(A) 2.0 V (B) 0.72 V (C) 0.48 V (D) 2.48 V

Sol. Given electric field is of a plane electromagnetic wave travelling in the x -direction. The expression for the electric field for these waves is

$$\vec{E}(x, t) = \vec{E}_0 \cos(2\pi x/\lambda - 2\pi\nu t).$$

Compare above equation with the given expression of electric field to get the wavelength $\lambda = 5 \times 10^{-7} \text{ m} = 5000 \text{ \AA}$ and the frequency $\nu = 6 \times 10^{14} \text{ Hz}$. Note that $c = \nu\lambda = 3 \times 10^8 \text{ m/s}$.

The maximum kinetic energy of the photoelectrons is given by

$$K_{\max} = \frac{hc}{\lambda} - \phi = \frac{12400 \text{ \AA} \cdot \text{eV}}{5000 \text{ \AA}} - 2 \text{ eV} = 0.48 \text{ eV}.$$

The stopping potential is $V_0 = K_{\max}/e = 0.48 \text{ V}$.

Ans. C \square

Q 39. The maximum kinetic energy of photoelectrons liberated from the surface of lithium ($\phi = 2.39 \text{ eV}$) by electromagnetic radiation whose electric field at the origin varies with time as

$$\vec{E} = \vec{E}_0 [1 + \cos(6 \times 10^{14}t)] \cos(42 \times 10^{14}t).$$

(A) 0.17 eV (B) 0.38 eV (C) 0.78 eV (D) 1.44 eV

Sol. The electric field at the origin varies with time as

$$\begin{aligned} \vec{E} &= \vec{E}_0(1 + \cos \omega_1 t) \cos \omega_2 t \\ &= \vec{E}_0(\cos \omega_2 t + \cos \omega_1 t \cos \omega_2 t) \\ &= \vec{E}_0 \cos \omega_2 t + \frac{\vec{E}_0}{2} \cos(\omega_1 + \omega_2)t + \frac{\vec{E}_0}{2} \cos(\omega_2 - \omega_1)t. \end{aligned}$$

It is the superposition of three waves of frequencies

$$\nu_1 = \frac{\omega_2 - \omega_1}{2\pi} = \frac{42 - 6}{2\pi} \times 10^{14} = 5.73 \times 10^{14} \text{ Hz,}$$

$$\nu_2 = \frac{\omega_2}{2\pi} = \frac{42}{2\pi} \times 10^{14} = 6.69 \times 10^{14} \text{ Hz,}$$

$$\nu_3 = \frac{\omega_1 + \omega_2}{2\pi} = \frac{6 + 42}{2\pi} \times 10^{14} = 7.64 \times 10^{14} \text{ Hz.}$$

Since ν_3 is the highest frequency, the photons of this frequency are most energetic. These photons eject electrons with the maximum kinetic energy

$$\begin{aligned} K_{\max} &= h\nu_3 - \phi \\ &= \frac{(6.63 \times 10^{-34})(7.64 \times 10^{14})}{1.6 \times 10^{-19}} - 2.39 \\ &= 3.17 - 2.39 = 0.78 \text{ eV.} \end{aligned}$$

We encourage you to compare the contributions of ν_1 , ν_2 , and ν_3 on the photocurrent. It is interesting and challenging, give it a try!

Ans. C \square

Q 40. The stopping potential for photoelectrons emitted from a surface illuminated by light of wavelength $\lambda = 4910 \text{ \AA}$ is 0.71 V. When the incident wavelength is changed the stopping potential is found to be 1.43 V. The new wavelength is
(A) 2437 \AA (B) 3820 \AA (C) 1910 \AA (D) 9889 \AA

Sol. The maximum kinetic energy of photoelectrons is related to the stopping potential V_0 by $K_{\max} = eV_0$.

Thus, the maximum kinetic energy of photoelectrons when surface is illuminated by light of wavelength $\lambda = 4910 \text{ \AA}$ is 0.71 eV. Its value is 1.43 eV for the new wavelength λ' . Substitute these values in equation $K_{\max} = hc/\lambda - \phi$ to get

$$0.71 \text{ eV} = \frac{hc}{\lambda} - \phi = \frac{12400 \text{ \AA} \cdot \text{eV}}{4910 \text{ \AA}} - \phi, \quad (1)$$

$$1.43 \text{ eV} = \frac{hc}{\lambda'} - \phi = \frac{12400 \text{ \AA} \cdot \text{eV}}{\lambda' \text{ \AA}} - \phi. \quad (2)$$

Subtract equation (1) from (2) and solve to get the new wavelength $\lambda' = 3820 \text{ \AA}$.

Ans. B \square

Q 41. When a certain photosensitive surface is illuminated with monochromatic light of frequency ν , the stopping potential for the photo current is $-V_0/2$. When the surface is illuminated by monochromatic light of frequency $\nu/2$, the stopping potential is $-V_0$. The threshold frequency for photoelectric emission is

(2019 m)

- (A) $\frac{5}{3}\nu$ (B) $\frac{4}{3}\nu$ (C) 2ν (D) $\frac{3}{2}\nu$

Sol. The stopping potential is the negative potential applied to the collector (relative to the emitter) to stop the maximum kinetic energy photoelectrons from reaching the collector. The maximum kinetic energy of photoelectrons is related to the stopping potential V_0 by $K_{\max} = eV_0$.

The maximum kinetic energy of the photoelectrons when the surface is illuminated by light of frequency ν

is $eV_0/2$ and the maximum kinetic energy of photoelectrons is eV_0 when the surface is illuminated by light of frequency $\nu/2$. Can you smell something fishy? The maximum kinetic energy of photoelectrons decreased when frequency is increased!

The threshold (or cutoff) frequency ν_c is the minimum frequency that can eject photoelectrons. It is related to the work function as $\phi = h\nu_c$.

Let's substitute given values in $K_{\max} = h\nu - \phi$ to get

$$eV_0/2 = h\nu - h\nu_c, \quad (1)$$

$$eV_0 = h\nu/2 - h\nu_c. \quad (2)$$

Divide equation (1) by (2) and solve to get $\nu_c = 3\nu/2$.

Now, you can clearly see the problem. The light of frequency ν or $\nu/2$ cannot eject photoelectrons. In these cases, the stopping potential does not make sense as electrons are not even ejected. Sometimes, questions are framed with a mechanical mindset without giving any thought about the physics principles.

Ans. See solution \square

Q 42. When radiation of wavelength λ is used to illuminate a metallic surface, the stopping potential is V_0 . When the same surface is illuminated with radiation of wavelength 3λ , the stopping potential is $V_0/4$. If the threshold wavelength for the metallic surface is $n\lambda$ then value of n will be (2020 m)

(A) 3 (B) 5 (C) 7 (D) 9

Sol. The maximum kinetic energy of photoelectrons is related to the stopping potential V_0 by $K_{\max} = eV_0$. The maximum kinetic energy of the photoelectrons when the surface is illuminated by radiation of wavelength λ is eV_0 and its value is $eV_0/4$ when the surface is illuminated by radiation of wavelength 3λ .

The threshold (or cutoff) wavelength λ_c is the maximum wavelength that can eject photoelectrons. It is related to the work function ϕ as $\phi = hc/\lambda_c$.

Substitute given values in equation $K_{\max} = hc/\lambda - \phi$ to get

$$eV_0 = hc/\lambda - hc/\lambda_c, \quad (1)$$

$$eV_0/4 = hc/(3\lambda) - hc/\lambda_c. \quad (2)$$

Divide equation (1) by (2) and solve to get $\lambda_c = 9\lambda$.

Ans. D \square

Q 43. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to (2019 m)

(A) 0.5 V (B) 1.5 V (C) 1.0 V (D) 2.0 V

Sol. Let V_1 and V_2 be the stopping potentials for the wavelengths $\lambda_1 = 300$ nm and $\lambda_2 = 400$ nm, respectively. Substitute these values in $K_{\max} = eV_0 =$

$hc/\lambda - \phi$ to get

$$eV_1 = \frac{hc}{\lambda_1} - \phi = \frac{1240 \text{ nm} \cdot \text{eV}}{300 \text{ nm}} - \phi, \quad (1)$$

$$eV_2 = \frac{hc}{\lambda_2} - \phi = \frac{1240 \text{ nm} \cdot \text{eV}}{400 \text{ nm}} - \phi. \quad (2)$$

Subtract equation (2) from (1) and solve to get the decrease in potential $V_1 - V_2 = 31/30 \approx 1 \text{ V}$. Note that the stopping potential decreases when the wavelength is increased. We encourage you to show that $\Delta V_0/V_0 = -\Delta\lambda/\lambda$ for small change in the wavelength.

Ans. C \square

Q 44. The ultraviolet light of wavelength 200 nm falls on an aluminium surface ($\phi = 4.2 \text{ eV}$). Which of the following option(s) is (are) correct?

- (A) The kinetic energy of the fastest emitted photoelectrons is 2.0 eV.
- (B) The kinetic energy of the slowest emitted photoelectrons is zero.
- (C) The stopping potential is 2.0 V.
- (D) The cut-off wavelength is 295 nm.

Sol. The energy of an ultraviolet photon of wavelength $\lambda = 200 \text{ nm}$ is

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ nm} \cdot \text{eV}}{200 \text{ nm}} = 6.2 \text{ eV}.$$

The fastest emitted photoelectron has the maximum kinetic energy given by

$$K_{\max} = E - \phi = 6.2 - 4.2 = 2.0 \text{ eV}.$$

The minimum kinetic energy of the emitted photoelectrons is $K_{\min} = 0$.

The stopping potential V_0 is related to the maximum kinetic energy of the photoelectron by $K_{\max} = eV_0$. Thus, the stopping potential is

$$V_0 = K_{\max}/e = 2.0 \text{ eV}/e = 2.0 \text{ V}.$$

The cutoff wavelength λ_c is related to the work function by $\phi = hc/\lambda_c$. Thus, the cutoff wavelength for the aluminium is

$$\lambda_c = \frac{hc}{\phi} = \frac{1240 \text{ nm} \cdot \text{eV}}{4.2 \text{ eV}} = 295 \text{ nm}.$$

UV light covers a wavelength spectrum from 100 nm to 400 nm. It is subdivided into three regions as (i) UVA from wavelength 315 nm to 400 nm, (ii) UVB from 280 nm to 315 nm and (iii) UVC from 100 nm to 280 nm. Can you see why UVC lamp is used in photoelectric effect demonstration from an aluminium plate?

Ans. A, B, C, D \square

Graph Based

Q 45. The work functions of silver and sodium are 4.6 eV and 2.3 eV, respectively. The ratio of the slope of the stopping potential *versus* frequency plot for silver to that of sodium is

(2013)

Sol. In photoelectric effect, the stopping potential (V_0) is related to the frequency (ν) of incident radiation and work function (ϕ) of the material by

$$V_0 = \frac{h}{e} \nu - \frac{\phi}{e}.$$

Thus, the slope of V_0 versus ν plot is h/e , a universal constant.

Ans. 1 \square

Q 46. According to Einstein's photoelectric equation, the plot of the kinetic energy of the emitted photoelectrons from a metal versus the frequency of the incident radiation is a straight line whose slope (2004 m)

- (A) depends on the nature of the metal used.
- (B) depends on the intensity of the radiation.
- (C) depends both on the intensity of the radiation and the metal used.
- (D) is the same for all metals and independent of the intensity of the radiation.

Sol. In photoelectric effect, the stopping potential V_0 is related to the frequency ν of incident radiation and work function ϕ of the material by

$$V_0 = \frac{h}{e} \nu - \frac{\phi}{e}.$$

Thus, the slope of V_0 versus ν plot is h/e , is a universal constant. It is the same for all metals and independent of the intensity of the radiation.

Ans. D \square

Q 47. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below:

λ (μm)	V_0 (Volt)
0.3	2.0
0.4	1.0
0.5	0.4

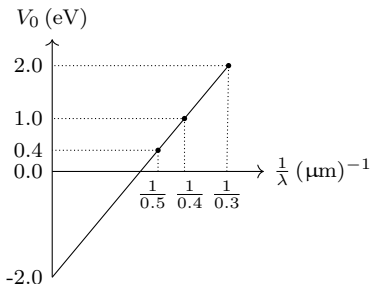
Given that $c = 3 \times 10^8$ m/s and $e = 1.6 \times 10^{-19}$ C, Planck's constant (in units of $\text{J} \cdot \text{s}$) found from such an experiment is (2016)

- (A) 6.0×10^{-34} (B) 6.4×10^{-34}
(C) 6.6×10^{-34} (D) 6.8×10^{-34}

Sol. In photoelectric effect, the stopping potential (V_0) is related to the wavelength of incident radiation by

$$eV_0 = hc/\lambda - \phi, \quad (1)$$

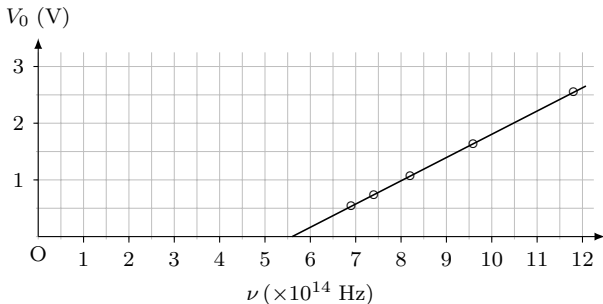
where ϕ is the work function of the metal. From equation (1), $1/\lambda$ versus V_0 graph is a straight line with a slope hc/e and intercept $-\phi/e$ on V_0 axis.



Substitute the values of V_0 and λ in equation (1) to get three equations in two unknowns, hc/e and ϕ/e . Solve any two of the three equations to get $h = 6.4 \times 10^{-34} \text{ J}\cdot\text{s}$ and $\phi = 2.0 \text{ eV}$. We encourage you to find h and ϕ if second reading is erroneously measured as $(0.4 \mu\text{m}, 1.1 \text{ V})$ instead of $(0.4 \mu\text{m}, 1.0 \text{ V})$.

Ans. B \square

Q 48. A historical experiment was done by R. Millikan to disprove Einstein's equation $K_{\text{max}} = h\nu - \phi$. His experimental results are shown in the figure. Which of the following option(s) is (are) correct?



- (A) The experimental results are in agreement with Einstein's equation.
- (B) The Planck's constant deduced from the graph is $h = 6.4 \times 10^{-34} \text{ J} \cdot \text{s}$.
- (C) The work function deduced from the graph is $\phi = 2.24 \text{ eV}$.
- (D) The graph intersect V_0 -axis at $V_0 = -2.24 \text{ V}$.

Sol. The relation between frequency ν and stopping potential V_0 is a straight line. The line intersect the frequency axis at $\nu_c = 5.6 \times 10^{14} \text{ Hz}$. It passes through the points $(8.0 \times 10^{14}, 1)$ and $(10.5 \times 10^{14}, 2)$ and its slope is

$$\begin{aligned}
 s &= \frac{\Delta V_0}{\Delta \nu} = \frac{2 - 1}{10.5 \times 10^{14} - 8 \times 10^{14}} \\
 &= 4 \times 10^{-15} \text{ V/Hz.}
 \end{aligned}$$

Thus, the equation of the line is

$$\begin{aligned}V_0 &= 4.0 \times 10^{-15}(\nu - 5.6 \times 10^{14}) \\ &= 4.0 \times 10^{-15}\nu - 2.24.\end{aligned}\quad (1)$$

Write Einstein's equation in terms of the stopping potential ($K_{\max} = eV_0$)

$$V_0 = (h/e)\nu - \phi/e. \quad (2)$$

Compare equations (1) and (2) to get Planck's constant and the work function

$$\begin{aligned}h &= es = (1.6 \times 10^{-19})(4.0 \times 10^{-15}) \\ &= 6.4 \times 10^{-34} \text{ J} \cdot \text{s}, \\ \phi &= e(2.24) \text{ J} = 2.24 \text{ eV}.\end{aligned}$$

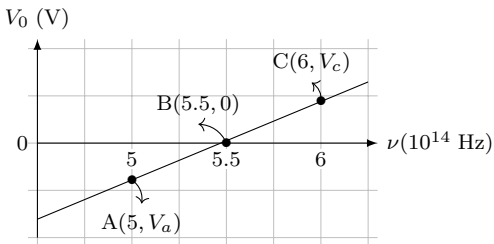
Substitute $\nu = 0$ in equation (1) to get intercept on the V_0 axis as $V_0 = -2.24 \text{ V}$. Note that the graph intersect the V_0 axis at $V_0 = -\phi/e$.

Albert Einstein was awarded Nobel Prize in 1921 for his services to theoretical physics, and especially for his discovery of the law of the photoelectric effect.

R. Millikan was awarded Nobel Prize in 1923 for his work on the elementary charge of electricity and on the photoelectric effect.

Ans. A, B, C, D \square

Q 49. Given figure shows a few data points in a photoelectric effect experiment for a certain metal. The minimum energy for ejection of electron from its surface is (2020 m)



- (A) 1.93 eV (B) 2.59 eV (C) 2.27 eV (D) 2.10 eV

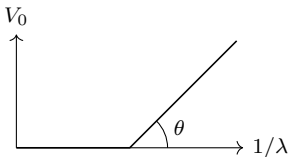
Sol. The stopping potential is zero at the threshold (or cutoff) frequency i.e., $V_0(\nu_c) = 0$. From given figure, the stopping potential is zero at the point B. The frequency at this point is $\nu_c = 5.5 \times 10^{14}$ Hz. The energy of a photon of this frequency is

$$\begin{aligned} E &= h\nu_c = (6.63 \times 10^{-34})(5.5 \times 10^{14}) \text{ J} \\ &= \frac{6.63 \times 10^{-34} \times 5.5 \times 10^{14}}{1.6 \times 10^{-19}} = 2.28 \text{ eV.} \end{aligned}$$

Thus, minimum energy for the ejection of photoelectrons is $\phi = h\nu_c = 2.28$ eV.

Ans. C \square

Q 50. In a photoelectric effect experiment, the graph of stopping potential V versus reciprocal of wavelength obtained is shown in the figure. As the intensity of incident radiation is increased (2020 m)



- (A) Straight line shifts to left
- (B) Straight line shift to right
- (C) Slope of the straight line get more steep
- (D) Graph does not change

Sol. The stopping potential V_0 is related to the wavelength λ of the incident radiation by

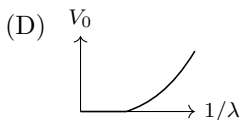
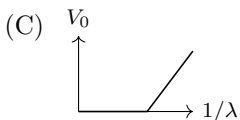
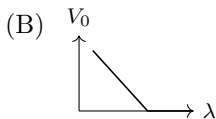
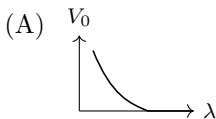
$$V_0 = \frac{hc}{e} \frac{1}{\lambda} - \frac{\phi}{e}.$$

The graph of V_0 versus $1/\lambda$ is a straight line with a constant slope hc/e and material dependent intercept ϕ/e . The graph does not depend on the intensity of the incident radiation.

Note: The intensity of incident radiation can be increased either by increasing the wave amplitude or by increasing the wave frequency. Here, we assumed that intensity is increased without varying the frequency.

Ans. D \square

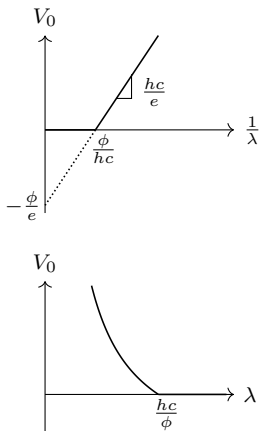
Q 51. For photoelectric effect with incident photon wavelength λ , the stopping potential is V_0 . Identify the correct variation(s) of V_0 with λ and $1/\lambda$. (2015)



Sol. In photoelectric effect, the stopping potential V_0 is related to the wavelength of incident radiation by

$$eV_0 = hc/\lambda - \phi, \quad (1)$$

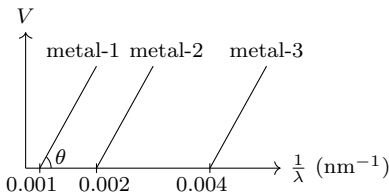
where ϕ is the work function of the metal. From equation (1), $1/\lambda$ versus V_0 graph is a straight line with a positive slope hc/e . The equation (1) can be written as $(eV_0 + \phi)\lambda = hc$.



Thus, λ *versus* V_0 graph is hyperbolic in nature. The details of two graphs are shown in the figures.

Ans. A, C \square

Q 52. The graph between $1/\lambda$ and stopping potential (V) of three metals having work functions ϕ_1 , ϕ_2 and ϕ_3 in an experiment of photoelectric effect is plotted as shown in the figure. Which of the following statement(s) is (are) correct? (Here λ is the wavelength of the incident ray.) (2006)



- (A) $\phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4$.
 (B) $\phi_1 : \phi_2 : \phi_3 = 4 : 2 : 1$.
 (C) $\tan \theta$ is directly proportional to hc/e , where h is Planck's constant and c is the speed of light.
 (D) The violet colour light can eject photoelectrons from metal 2 and 3.

Sol. The energy of incident photon hc/λ is equal to the sum of minimum energy required to emit the photoelectron (work function ϕ) and kinetic energy K of the ejected photoelectron. The stopping potential V converts the kinetic energy K of ejected photoelectron into potential energy eV thereby stopping its motion. Thus, $hc/\lambda = \phi + K = \phi + eV$ i.e.,

$$V = \frac{hc}{e} \frac{1}{\lambda} - \frac{\phi}{e}.$$

At $V = 0$, $\phi = hc/\lambda$, which gives

$$\phi_1 = 0.001hc, \quad \phi_2 = 0.002hc, \quad \phi_3 = 0.004hc.$$

Hence, $\phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4$. The slope of $V - \frac{1}{\lambda}$ graph (which is a straight line) is $\tan \theta = hc/e$. The

wavelengths corresponding to ϕ_1 , ϕ_2 and ϕ_3 are

$$\lambda_1 = 1/0.001 = 1000 \text{ nm,}$$

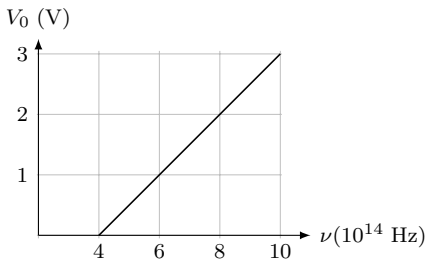
$$\lambda_2 = 1/0.002 = 500 \text{ nm,}$$

$$\lambda_3 = 1/0.004 = 250 \text{ nm.}$$

The wavelength of violet colour light is close to 400 nm. Thus, violet light cannot eject the photoelectrons from metal 3.

Ans. A, C \square

Q 53. The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be (2019 m)



(A) 1.82 eV (B) 1.66 eV (C) 1.95 eV (D) 2.12 eV

Sol. The work function ϕ is equal to the energy of a photon of cutoff frequency ν_c . The stopping potential is zero at the cutoff frequency i.e., $V_0(\nu_c) = 0$. From

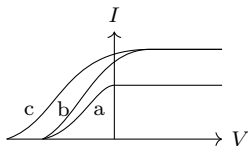
given figure, the cutoff frequency is $\nu_c = 4 \times 10^{14}$ Hz. Thus, the work function of the sodium is

$$\begin{aligned}\phi &= h\nu_c = (6.63 \times 10^{-34})(4 \times 10^{14}) \text{ J} \\ &= \frac{(6.63 \times 10^{-34})(4 \times 10^{14})}{1.6 \times 10^{-19}} \text{ eV} = 1.66 \text{ eV}.\end{aligned}$$

Ans. B \square

Photocurrent

Q 54. The figure shows the variation of photocurrent with anode potential for a photosensitive surface for three different radiations. Let I_a, I_b and I_c be the intensities and f_a, f_b and f_c be the frequencies for the curves a, b and c respectively. Then, (2004)



- (A) $f_a = f_b$ and $I_a \neq I_b$ (B) $f_a = f_c$ and $I_a = I_c$
 (C) $f_a = f_b$ and $I_a = I_b$ (D) $f_b = f_c$ and $I_b = I_c$

Sol. In photoelectric effect, the cut-off voltage depends on the frequency of incident radiation and saturation current depends on its intensity. The cut-off voltages are equal for a and b but different for c which gives

$f_a = f_b \neq f_c$. The saturation currents are equal for b and c but different for a which gives $I_a \neq I_b = I_c$.

Ans. A \square

Q 55. When a monochromatic point source of light is at a distance of 0.2 m from a photoelectric cell, the cut-off voltage and the saturation current are 0.6 V and 18.0 mA, respectively. If the same source is placed 0.6 m away from the photoelectric cell, then, (1992)

- (A) the stopping potential will be 0.2 V.
- (B) the stopping potential will be 0.6 V.
- (C) the saturation current will be 6.0 mA.
- (D) the saturation current will be 2.0 mA.

Sol. The intensity of a point source at a distance r varies as $I = I_0/r^2$, where I_0 is a constant. Thus, intensities of the light falling on the photoelectric cell are

$$I_1 = I_0/(0.2)^2, \quad I_2 = I_0/(0.6)^2.$$

Since saturation current is proportional to the intensity

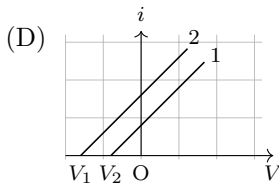
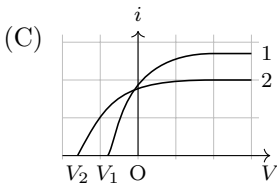
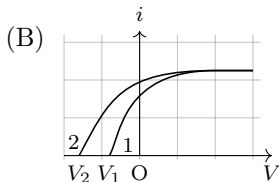
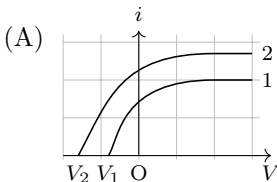
$$i_2 = \frac{I_2}{I_1} i_1 = \frac{(0.2)^2}{(0.6)^2} \times 18 = 2 \text{ mA}.$$

The cut-off voltage depends on the frequency of incident light and not on its intensity. Thus, the cut-off voltage is 0.6 V in both the cases.

Ans. B, D \square

Q 56. A metal surface is illuminated with a monochromatic radiation of frequency ν_1 and of the energy flux S .

The photocurrent i is measured for various values of applied potential V . The experiment is repeated with radiation of frequency $\nu_2 (> \nu_1)$ but of the same energy flux S . Which of the following graphs best represent the results of these experiments?



Sol. The stopping potential is related to the maximum kinetic energy of photoelectrons by

$$V_0 = \frac{K_{\max}}{e} = \frac{h}{e} \nu - \frac{\phi}{e}.$$

Since $\nu_2 > \nu_1$, we get stopping potential $|V_2| > |V_1|$.

The saturation current is proportional to the number of ejected photoelectrons, which in turn is proportional to the number of incident photons. The energy of a photon of frequency ν_1 is $E_1 = h\nu_1$. The photons flux (numbers of photons striking per unit area per unit

time) of this frequency is

$$n_1 = \frac{S}{E_1} = \frac{S}{h\nu_1}.$$

The energy of a photon of frequency ν_2 is $E_2 = h\nu_2$. The photons flux for this frequency is

$$n_2 = \frac{S}{E_2} = \frac{S}{h\nu_2}.$$

Since $\nu_2 > \nu_1$, we get $n_1 > n_2$. Hence, saturation current is more in case 1.

Note that equal energy flux (or intensity) does not mean equal photon flux (and hence saturation current). Many books state “saturation photocurrent increases with increase in the intensity of the radiation”. This is true only if frequency is constant.

Ans. C \square

Q 57. The frequency and the intensity of a beam of light falling on the surface of a photoelectric material are increased by a factor of two. This will

- (A) increase the maximum kinetic energy of ejected photoelectrons, as well as photocurrent by a factor of two.
- (B) increase the maximum kinetic energy of ejected photoelectrons and would increase photocurrent by a factor of two.
- (C) increase the maximum kinetic energy of ejected photoelectrons by a factor of two and have no effect on photocurrent.

- (D) increase the maximum kinetic energy of ejected photoelectron by more than twice and will have no effect on photocurrent.

Sol. The maximum kinetic energy of ejected photoelectrons is related to the frequency of incident radiation by $K_{\max} = h\nu - \phi$. Thus, the maximum kinetic energy of photoelectrons in two cases are

$$K_{\max,1} = h\nu - \phi, \quad (1)$$

$$K_{\max,2} = h(2\nu) - \phi. \quad (2)$$

Eliminate ν from equations (1) and (2) to get

$$K_{\max,2} = 2K_{\max,1} + \phi.$$

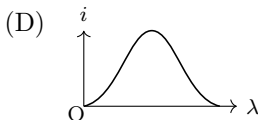
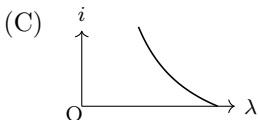
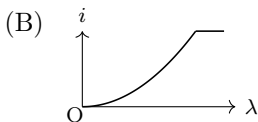
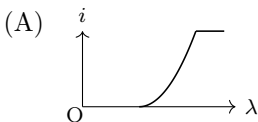
Since work function $\phi > 0$, the maximum kinetic energy of the photoelectrons is increased by more than twice.

The energy of a photon of frequency ν is $E_1 = h\nu$ and that of a photon of frequency 2ν is $E_2 = 2h\nu$. The photon-flux (number of photons striking per unit area per unit time normal to the surface) of the radiations of intensities $I_1 = I$ and $I_2 = 2I$ are

$$n_1 = \frac{I_1}{E_1} = \frac{I}{h\nu},$$
$$n_2 = \frac{I_2}{E_2} = \frac{2I}{2h\nu} = \frac{I}{h\nu}.$$

Thus, photon-flux remains same when the frequency and the intensity are both doubled. Thus, the saturation photocurrent will not change.

Q 58. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current i of the photocell varies as follows (2006 m)



Sol. The photocurrent i is proportional to the numbers of electrons ejected per unit time, which in turn is proportional to the numbers of photons incident per unit time. The number of photons incident per unit time depend on the intensity and wavelength of the incident light.

The energy of each photon is $E = hc/\lambda$. Let n be the number of photons incident per unit time per unit area. The energy incident per unit time per unit area (i.e., intensity I) is given by $I = nhc/\lambda$. Thus, the number of photons incident per unit time per unit area is

$$n = I\lambda/(hc).$$

The question is silent on the intensity I of the light. If the wavelength λ is varied without varying the intensity then n (and hence photocurrent i) shall vary linearly

with the wavelength. None of the given options show straight line dependence.

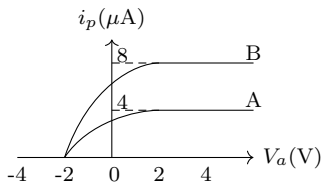
The photocurrent will become zero when the wavelength of light is increased beyond a threshold (or cut-off) wavelength λ_c . The photons of wavelength greater than λ_c cannot eject electrons because their energy is less than the work function.

Ans. See solution \square

Q 59. In a photoelectric experiment set-up, photons of energy 5 eV falls on the cathode having work function 3 eV. If the saturation current $i_A = 4 \mu\text{A}$ for intensity $I_A = 10^{-5} \text{ W/m}^2$, then plot the variation of photocurrent i_p against the anode voltage V_a for photon intensity $I_A = 10^{-5} \text{ W/m}^2$ and $I_B = 2 \times 10^{-5} \text{ W/m}^2$. (2003)

Sol. In photoelectric effect, maximum kinetic energy of the ejected photoelectrons is given by

$$K_{\max} = h\nu - \phi = 5 - 3 = 2 \text{ eV}.$$



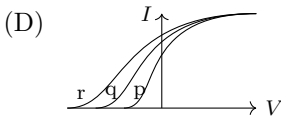
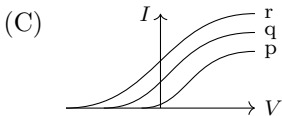
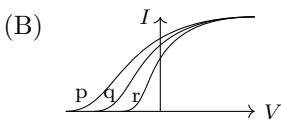
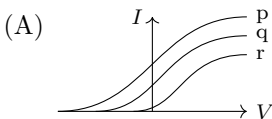
Thus, anode potential to stop these photoelectrons is 2 V (stopping potential). The saturation current is

proportional to the intensity of incident light. Thus,

$$i_B = \frac{I_B i_A}{I_A} = \frac{(2 \times 10^{-5})(4)}{10^{-5}} = 8 \mu\text{A}.$$

Ans. See solution \square

Q 60. Photoelectric effect experiments are performed using three metal plates p , q and r having work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I - V graph for the experiment is [Take $hc = 1240$ eV-nm.] (2009)



Sol. The photon energy for wavelength λ is given by

$$E = hc/\lambda. \quad (1)$$

Substitute the values in equation (1) to get the photon energies for three given wavelengths as

$$E_1 = 1240/550 = 2.25 \text{ eV},$$

$$E_2 = 1240/450 = 2.75 \text{ eV},$$

$$E_3 = 1240/350 = 3.54 \text{ eV}.$$

The energies E_1 , E_2 , and E_3 are sufficient to emit the photoelectrons from metal p , E_2 and E_3 are sufficient to emit the photoelectrons from metal q , and E_3 is sufficient to emit the photoelectrons from metal r . Thus, the saturation current, $I_p > I_q > I_r$. The stopping potential V is related to work function ϕ by

$$V = hc/(e\lambda) - \phi/e. \quad (2)$$

Since $\phi_p < \phi_q < \phi_r$, equation (2) gives the stopping potential $V_p > V_q > V_r$.

Ans. A \square

Q 61. A beam of light has three wavelengths 4144 Å, 4972 Å and 6216 Å with a total energy of 3.6×10^{-3} W/m² equally distributed amongst the three wavelengths. The beam falls normally on an area 1 cm² of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in two seconds. (1989)

Sol. The energies of the photons for wavelengths $\lambda_1 = 4144$ Å, $\lambda_2 = 4972$ Å, and $\lambda_3 = 6216$ Å are

$$e_1 = hc/\lambda_1 = 12420/4144 = 3.0 \text{ eV},$$

$$e_2 = hc/\lambda_2 = 12420/4972 = 2.5 \text{ eV},$$

$$e_3 = hc/\lambda_3 = 12420/6216 = 2.0 \text{ eV}.$$

A photon ejects the photoelectron from a metal of work function $\phi = 2.3$ eV if its energy is greater than

ϕ . Thus, only the photons of wavelengths λ_1 and λ_2 eject the photoelectrons from the given metal. Total intensity, $I = 3.6 \times 10^{-3} \text{ W/m}^2$, is equally divided among the three wavelengths. Thus, the intensities of the wavelengths λ_1 and λ_2 are, $I_1 = I_2 = I/3 = 1.2 \times 10^{-3} \text{ W/m}^2$. The energies falling on an area $A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$ for wavelengths λ_1 and λ_2 in time $t = 2 \text{ s}$ are

$$E_1 = I_1 A t = (1.2 \times 10^{-3})(10^{-4})(2) = 2.4 \times 10^{-7} \text{ J},$$

$$E_2 = I_2 A t = (1.2 \times 10^{-3})(10^{-4})(2) = 2.4 \times 10^{-7} \text{ J},$$

and the number of photons corresponding to these energies are

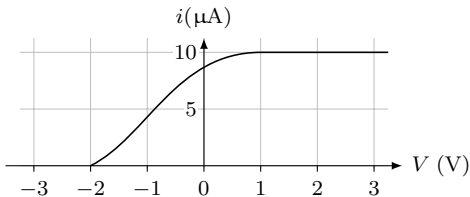
$$n_1 = \frac{E_1}{e_1} = \frac{2.4 \times 10^{-7}}{3.0 \times 1.602 \times 10^{-19}} = 5 \times 10^{11},$$

$$n_2 = \frac{E_2}{e_2} = \frac{2.4 \times 10^{-7}}{2.5 \times 1.602 \times 10^{-19}} = 6 \times 10^{11}.$$

Since each photon of wavelength λ_1 and λ_2 ejects one photoelectron, total number of photoelectrons ejected from the metal are $n_e = n_1 + n_2 = 1.1 \times 10^{12}$.

Ans. 1.1×10^{12} \square

Q 62. The results of a photoelectric effect experiment are shown in the figure. Which of the following is (are) correct?



- (A) The stopping potential is 2 V.
 (B) The saturation current is 10 μA .
 (C) The number of photoelectrons ejected per second is 6.25×10^{13} .
 (D) The wavelength of incident radiation is 620 nm.

Sol. The stopping potential is the negative potential applied to the collector that stops movement of photoelectrons. The photocurrent is zero at the stopping potential. Thus, stopping potential in this experiment is $V_0 = 2$ V. The maximum kinetic energy of photoelectrons is $K_{\text{max}} = eV_0 = 2$ eV.

The saturation current is the maximum value of photocurrent. It flows when the collector is given a sufficiently high positive potential. The saturation current in this experiment is $i_s = 10$ μA .

At saturation, all ejected photoelectrons reach the collector. Thus, the number of electrons ejected per second is equal to the number of electrons flowing in the circuit per second, which is

$$n = \frac{i_s}{e} = \frac{10 \times 10^{-6}}{1.6 \times 10^{-19}} = 6.25 \times 10^{13}.$$

We cannot find the wavelength of incident radiation and work function of the material from the given data.

We encourage you to analyse the phenomenon when applied voltage is increased from -2 V (stopping potential) to 1 V (saturation). Why the curve has a slow rise and not a steep rise? Try to correlate this with the velocity distribution of ejected photoelectrons.

Can you explain why the photocurrent at $V = 0$ is less than the saturation current i.e., why $i(V = 0) < i_s$? *Hint:* Think about the direction in which photoelectrons are ejected, geometrical shape and distance of the collector plate, and the role played by the electric field that exist between two plates.

Ans. A, B, C \square

Q 63. A metal cathode of work function 3.3 eV is illuminated with 15 μ W of light having a wavelength of 300 nm. The efficiency of converting photons to photoelectrons is 12% . The current measured in the experiment is

- (A) 43.5 μ A (B) 4.35 μ A
(C) 362.5 μ A (D) 36.25 μ A

Sol. The energy of a photon of wavelength 300 nm is

$$E = hc/\lambda = 1240/300 = 4.13 \text{ eV.}$$

The photon energy is greater than the work function $\phi = 3.3$ eV. It can eject an electron from the metal.

The incident power is $P = 15 \times 10^{-6}$ W. The number of photons incident per second is

$$n_p = \frac{P}{E} = \frac{15 \times 10^{-6}}{4.13 \times 1.6 \times 10^{-19}} = 2.27 \times 10^{13}.$$

The efficiency of converting photons to photoelectrons is $\eta = 12\%$. Thus, the number of electrons ejected per second is

$$n_e = \eta n_p = (0.12)(2.27 \times 10^{13}) = 2.72 \times 10^{12}.$$

The current measured in the experiment (photocurrent) i.e., charge flowing per second is given by

$$\begin{aligned} i &= en_e = (1.6 \times 10^{-19})(2.72 \times 10^{12}) \\ &= 4.35 \times 10^{-7} \text{ A} = 43.5 \text{ } \mu\text{A}. \end{aligned}$$

Ans. A \square

Photoelectric Effect: Miscellaneous

Q 64. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^Z$ (where $1 < A < 10$). The value of Z is
(2011)

Sol. The silver sphere gets positively charged due to emission of photoelectrons. This positively charged sphere attracts (binds) the emitted photoelectrons. The emitted photoelectrons cannot escape if their kinetic energies ($hc/\lambda - \phi$) are less than or equal to their potential energies ($\frac{1}{4\pi\epsilon_0} \frac{ne^2}{r}$). Thus, in limiting case,

$$\frac{hc}{\lambda} - \phi = \frac{1}{4\pi\epsilon_0} \frac{ne^2}{r}. \quad (1)$$

Substitute the values of various parameters in equation (1),

$$\frac{1242}{200} - 4.7 = \frac{n(9 \times 10^9)(1.6 \times 10^{-19})}{10^{-2}},$$

to get $n = 1.04 \times 10^7$. [We have used $hc = 1242$ eV-nm.]

Exercise: Up to what maximum potential an isolated copper sphere ($\phi = 4.47$ eV) can be charged when irradiated by electromagnetic radiation of wavelength $\lambda = 140$ nm? *Ans.* 4.39 eV.

Ans. 7 \square

Q 65. Two metallic plates A and B each of area 5×10^{-4} m², are placed parallel to each other at separation of 1 cm. Plate B carries a positive charge of 33.7×10^{-12} C. A monochromatic beam of light, with photons of energy 5 eV each, starts falling on plate A at $t = 0$ so that 10^{16} photons fall on it per square metre per second. Assume that one photoelectron is emitted for every 10^6 incident photons. Also assume that all the emitted photoelectrons are collected by plate B and the

work function of plate A remains constant at the value of 2 eV. Determine, (2002)

- the number of photoelectrons emitted up to $t = 10$ s.
- the magnitude of the electric field between the plates A and B at $t = 10$ s.
- The kinetic energy of the most energetic photoelectrons emitted at $t = 10$ s when it reaches plate B .

[Neglect time taken by the photoelectron to reach plate B . Take $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$]

Sol. The number of photons striking the plate A of area $A = 5 \times 10^{-4} \text{ m}^2$ in a time interval of 10 s is

$$n_p = (10^{16}) (5 \times 10^{-4}) (10) = 5 \times 10^{13},$$

and the number photoelectrons emitted by this plate in 10 s is

$$n_e = n_p/10^6 = 5 \times 10^7.$$

The positive charge on plate A due to emission of the photoelectrons is

$$q_A = en_e = (1.6 \times 10^{-19})(5 \times 10^7) = 8.0 \times 10^{-12} \text{ C},$$

and, the charge on plate B after collecting n_e photoelectrons is

$$q_B = 33.7 \times 10^{-12} - 8.0 \times 10^{-12} = 25.7 \times 10^{-12} \text{ C}.$$

The direction of electric field between the two plates is from plate B to plate A . The magnitude of this electric

field is

$$\begin{aligned}
 E &= E_B - E_A = \frac{q_B/A}{2\epsilon_0} - \frac{q_A/A}{2\epsilon_0} = \frac{q_B - q_A}{2A\epsilon_0} \\
 &= \frac{(25.7 - 8) \times 10^{-12}}{2(5 \times 10^{-4})(8.85 \times 10^{-12})} = 2 \times 10^3 \text{ V/m.}
 \end{aligned}$$

In photoelectric effect, the maximum kinetic energy of emitted photoelectrons is given by

$$K_{\max} = h\nu - \phi = 5 - 2 = 3 \text{ eV.}$$

These electrons are attracted by plate B. The gain in potential energy of the photoelectron is

$$U = qEd = eEd = e(2 \times 10^3)(10^{-2}) = 20 \text{ eV.}$$

Thus, kinetic energy of the most energetic photoelectrons when these reach plate B is $K_{\max} + U = 23 \text{ eV}$.

Ans. (a) 5×10^7 (b) $2 \times 10^3 \text{ N/C}$ (c) 23 eV \square

Q 66. In a photoelectric experiment a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100%. A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4} \text{ N}$ due to impact of the

electrons. The value of n is [Mass of the electron $m_e = 9 \times 10^{-31}$ kg and $1.0 \text{ eV} = 1.6 \times 10^{-19}$ J.]
(2018)

Sol. The frequency of incident light is just above the threshold frequency. Hence, the energy of each photon is equal to the work function ($E_p = \phi = 6.25 \text{ eV}$) and the kinetic energy of emitted photo-electron is zero ($K_{e0} = 0$). The energy incident per second on the cathode is incident power $P = 200 \text{ W}$. Thus, number of photons incident per second is

$$N_p = P/E_p = P/\phi.$$

The photo-electron emission efficiency is 100%. Thus, number of photo-electron emitted per second is equal to the number of photons incident per second i.e., $N_e = N_p$. These photo-electrons are accelerated by a potential difference $V = 500 \text{ V}$. Thus, gain in potential energy of each photo-electron is $\Delta U = eV = 500 \text{ eV}$. By conservation of energy, kinetic energy of the photo-electron when it reaches the anode is

$$K_e = K_{e0} + \Delta U = 500 \text{ eV}.$$

The linear momentum of photo-electron of mass m_e and kinetic energy K_e is given by

$$p_e = \sqrt{2m_e K_e}.$$

The photo-electron transfer its entire linear momentum to the anode (absorbed by the anode). Thus, gain in

linear momentum of the anode by absorbing one photoelectron is

$$\Delta p_a = p_e.$$

The force on the anode is equal to the increase in its linear momentum per second. Since N_e photo-electrons strikes the anode per second, the force acting on the anode is given by

$$\begin{aligned} F_a &= N_e \Delta p_a = N_e p_e && (\because \Delta p_a = p_e) \\ &= N_p p_e = (P/\phi) p_e && (\because N_e = N_p = P/\phi) \\ &= (P/\phi) \sqrt{2m_e K_e} && (\because p_e = \sqrt{2m_e K_e}) \\ &= (P/\phi) \sqrt{2m_e eV} && (\because K_e = eV) \\ &= 24 \times 10^{-4} \text{ N.} \end{aligned}$$

Ans. 24 \square

Q 67. When a beam of 10.6 eV photons of intensity 2.0 W/m^2 falls on a platinum surface of area $1.0 \times 10^{-4} \text{ m}^2$ and work function 5.6 eV, 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies (in eV). [Take $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$.] (2000)

Sol. The energy of each photon is

$$\begin{aligned} h\nu &= 10.6 \text{ eV} = 10.6 \times 1.6 \times 10^{-19} \\ &= 16.96 \times 10^{-19} \text{ J.} \end{aligned} \tag{1}$$

The intensity $I = 2 \text{ W/m}^2$ is the energy falling on 1 m^2 area in one second. Thus, the energy of photons falling on area $A = 1.0 \times 10^{-4} \text{ m}^2$ in one second is

$$E = IA = 2 \times 1.0 \times 10^{-4} = 2 \times 10^{-4} \text{ J/s.} \quad (2)$$

The equations (1) and (2) give the number of photons falling in one second as

$$n_p = \frac{E}{h\nu} = \frac{2 \times 10^{-4}}{16.96 \times 10^{-19}} = 1.18 \times 10^{14}.$$

The number of photoelectrons emitted in one second is 0.53% of n_p i.e.,

$$n_e = n_p \times 0.53/100 = 6.25 \times 10^{11}.$$

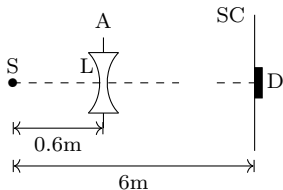
Maximum kinetic energy of emitted photoelectrons is related to work function ϕ by $h\nu = \phi + K_{\max}$,

$$K_{\max} = h\nu - \phi = 10.6 - 5.6 = 5.0 \text{ eV.}$$

Minimum kinetic energy of photoelectrons can be zero i.e., $K_{\min} = 0$.

Ans. 6.25×10^{11} , zero, 5.0 eV \square

Q 68. A monochromatic point source S radiating wavelength 6000 \AA with power 2 W , an aperture A of diameter 0.1 m and a large screen SC are placed as shown in the figure. A photoemissive detector D of surface area 0.5 cm^2 is placed at the centre of the screen. The efficiency of the detector for the photoelectron generation per incident photon is 0.9 . (1991)



- (a) Calculate the photon flux at the centre of the screen and photocurrent in the detector.
- (b) If the concave lens L of focal length 0.6 m is inserted in the aperture as shown, find the new values of photon flux and photocurrent. Assume a uniform average transmission of 80% from the lens.
- (c) If the work function of the photoemissive surface is 1 eV , calculate the values of the stopping potential in the two cases (without and with the lens in aperture).

Sol. The energy of each photon is

$$\begin{aligned}
 E &= \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}) (3 \times 10^8)}{6000 \times 10^{-10}} \\
 &= 3.315 \times 10^{-19}\text{ J}.
 \end{aligned}$$

The number of photons emitted per second by the source of power $P = 2\text{ W}$ is

$$N_0 = \frac{P}{E} = \frac{2}{3.315 \times 10^{-19}} = 6.03 \times 10^{18}.$$

The aperture does not obstruct radiations falling on the detector. Also, aperture size is much larger than

the wavelength. Hence, presence of aperture does not change the number of photons falling on the detector i.e., we can neglect diffraction effects.

The photon flux (number of photons per unit area per unit time) at the detector placed at a distance $r = 6$ m from the source S is

$$n = \frac{N_0}{4\pi r^2} = \frac{6.03 \times 10^{18}}{4(3.14)(6)^2} = 1.33 \times 10^{16}.$$

The number of photons incident on the detector of area A per second is

$$\begin{aligned} N_D &= nA = (1.33 \times 10^{16})(0.5 \times 10^{-4}) \\ &= 6.67 \times 10^{11}. \end{aligned}$$

The number of photoelectron emitted per second with a detector efficiency $\eta = 0.9$ is

$$N_e = \eta N_D = 6.00 \times 10^{11}.$$

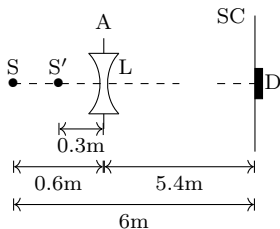
The photocurrent is charge of emitted photoelectrons per second i.e.,

$$\begin{aligned} i &= eN_e = 1.6 \times 10^{-19} \times 6.00 \times 10^{11} \\ &= 9.60 \times 10^{-8} \text{ A}. \end{aligned}$$

Now, a concave lens of focal length 0.6 m is inserted in the aperture. It diverges the beam. An observer on the screen see the beam as if it is coming from a new point source S' (image of S). The lens formula,

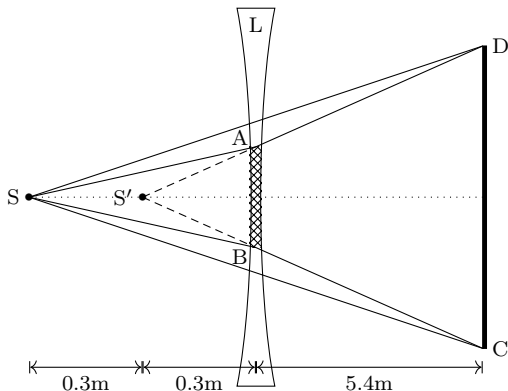
$$1/v - 1/u = 1/f,$$

with $u = -0.6$ m and $f = -0.6$ m gives position of S' as $v = -0.3$ m.



Let us focus on the radiations coming to the detector. The situation is shown in the figure given below (not to the scale). The radiations falling on the portion AB of the lens reaches the detector CD. From triangles $S'AB$ and $S'CD$, length $AB = CD \times 0.3/5.7$. Thus, for a symmetrical detector of area A , the area of shaded portion AB of the lens is

$$\begin{aligned} A_L &= A(0.3/5.7)^2 = 0.5 \times 10^{-4}(0.3/5.7)^2 \\ &= 1.39 \times 10^{-7} \text{ m}^2. \end{aligned}$$



The incident flux on the lens placed at a distance $r_1 = 0.6$ m from the source S is $N_0/(4\pi r_1^2)$. Thus, number of photons reaching the shaded portion of the lens (and hence reaching the detector) is

$$N_L = \frac{N_0}{4\pi r_1^2} \cdot A_L = \frac{6.03 \times 10^{18} \times 1.39 \times 10^{-7}}{4 \times 3.14(0.6)^2} = 1.85 \times 10^{11}.$$

The number of photons transmitted by the lens per second that reaches the detector is

$$N'_D = TN_L = 0.8 \times 1.85 \times 10^{11} = 1.48 \times 10^{11}.$$

The photon flux at the detector is

$$n' = \frac{N'_D}{A} = \frac{1.48 \times 10^{11}}{0.5 \times 10^{-4}} = 2.96 \times 10^{15}.$$

The number of electrons emitted per second by the detector is

$$N'_e = \eta N'_D = 1.33 \times 10^{11},$$

and the photocurrent is

$$i' = eN'_e = 2.13 \times 10^{-8} \text{ A.}$$

The maximum kinetic energy of emitted photoelectrons is given by

$$K_{\max} = \frac{hc}{\lambda} - \phi = \frac{3.315 \times 10^{-19}}{1.6 \times 10^{-19}} - 1 = 1.06 \text{ eV.}$$

Thus, the stopping potential is 1.06 V. It is same in both the cases.

We encourage you to find the illuminated area of the screen (i) with aperture and (ii) with lens. The lens increases illuminated area by almost 4 times and hence other quantities reduces in the same proportion.

- Ans.* (a) $1.33 \times 10^{16} \text{ s}^{-1}\text{m}^{-2}$, $9.60 \times 10^{-8} \text{ A}$
(b) $2.96 \times 10^{15} \text{ s}^{-1}\text{m}^{-2}$, $2.13 \times 10^{-8} \text{ A}$
(c) 1.06 V in both cases. \square

Q 69. A beam of electromagnetic radiation of intensity $6.4 \times 10^{-5} \text{ W/cm}^2$ is comprised of wavelength $\lambda = 310 \text{ nm}$. It falls normally on a metal (work function $\phi = 2 \text{ eV}$) of surface area 1 cm^2 . If one in 10^3 photons ejects an electron, total number of electrons ejected in 1 s is 10^x , then x is..... (2020 m)

- (A) 5 (B) 7 (C) 9 (D) 11

Sol. The energy falling on the metal surface of area $A = 1 \text{ cm}^2$ in one-second is

$$P = IA = (6.4 \times 10^{-5})(1) = 6.4 \times 10^{-5} \text{ W.}$$

The energy of a photon of wavelength $\lambda = 310 \text{ nm}$ is

$$\begin{aligned} E &= \frac{hc}{\lambda} = \frac{1240 \text{ nm} \cdot \text{eV}}{310 \text{ nm}} = 4 \text{ eV} \\ &= 4 \times 1.6 \times 10^{-19} = 6.4 \times 10^{-19} \text{ J.} \end{aligned}$$

The number of photons falling per second on the metal surface is

$$n_p = \frac{P}{E} = \frac{6.4 \times 10^{-5}}{6.4 \times 10^{-19}} = 10^{14}.$$

The energy of each photon is greater than the work function $\phi = 2 \text{ eV}$. Since one in 10^3 photons eject an electron, the number of photoelectrons ejected in one second is

$$n_e = \frac{n_p}{10^3} = \frac{10^{14}}{10^3} = 10^{11}.$$

Ans. D \square

Q 70. A metal plate of area $1 \times 10^{-4} \text{ m}^2$ is illuminated by a radiation of intensity 16 mW/m^2 . The work function of the metal is 5 eV . The energy of the incident photons is 10 eV and only 10% of it produces photoelectrons. The number of emitted photoelectrons per second and their maximum energy, respectively, will be

(2019 m)

- (A) 10^{14} and 10 eV (B) 10^{12} and 5 eV
(C) 10^{11} and 5 eV (D) 10^{10} and 5 eV

Sol. The energy falling on the metal plate of area $A = 10^{-4} \text{ m}^2$ in one-second is

$$P = IA = (16 \times 10^{-3})(10^{-4}) = 1.6 \times 10^{-6} \text{ J.}$$

The energy of each photon is $E = 10 \text{ eV}$. The number of photons falling per second on the plate is

$$n_p = \frac{P}{E} = \frac{1.6 \times 10^{-6}}{10 \times 1.6 \times 10^{-19}} = 10^{12}.$$

Since 10% of the photons eject the photoelectrons, the number of electrons ejected per second is

$$n_e = 0.1n_p = 0.1(10^{12}) = 10^{11}.$$

The energy of each photon is more than the work function of the metal $\phi = 5 \text{ eV}$. The maximum kinetic energy of the photoelectrons is

$$K_{\max} = E - \phi = 10 - 5 = 5 \text{ eV.}$$

Ans. C \square

Q 71. Radiation of wavelength $\lambda = 310 \text{ nm}$ falls on a photocell operating in the saturation mode. The sensitivity of the photocell is 4.8 mA/W . The number of photoelectrons produced by each incident photon is
(A) 0.003 (B) 0.02 (C) 0.03 (D) 0.2

Sol. The sensitivity of photocell is 4.8 mA/W . It means a current $i = 4.8 \text{ mA}$ flows for every 1 W of

incident radiation. The number of electrons ejected per second for 1 W of incident radiation is

$$n_e = \frac{i}{e} = \frac{4.8 \times 10^{-3}}{1.6 \times 10^{-19}} = 3 \times 10^{16}.$$

The energy of a photon of wavelength $\lambda = 310$ nm is

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ nm} \cdot \text{eV}}{310 \text{ nm}} = 4 \text{ eV} = 6.4 \times 10^{-19} \text{ J}.$$

The number of photons incident per second to get a radiation power $P = 1$ W is

$$n_p = \frac{P}{E} = \frac{1}{6.4 \times 10^{-19}} = 1.56 \times 10^{18}.$$

Thus, the number of photoelectrons produced by each incident photon is

$$\eta = \frac{n_e}{n_p} = \frac{3 \times 10^{16}}{1.56 \times 10^{18}} \approx 0.02.$$

Ans. B \square

Q 72. Radiation, with wavelength 6561 \AA falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of $3 \times 10^{-4} \text{ T}$. If the radius of the largest circular path followed by the electrons is 10 mm, the work function of the metal is close to (2020 m)

(A) 0.8 eV (B) 1.1 eV (C) 1.8 eV (D) 1.6 eV

Sol. The maximum kinetic energy of photoelectrons ejected by photons of wavelength $\lambda = 6561 \text{ \AA}$ from a metal of work function ϕ is

$$K_{\max} = hc/\lambda - \phi. \quad (1)$$

The maximum kinetic energy photoelectrons have the maximum linear momentum given by

$$mv = \sqrt{2mK_{\max}}. \quad (2)$$

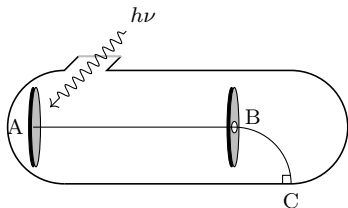
In a uniform magnetic field $B = 3 \times 10^{-4} \text{ T}$, the electrons with the maximum momentum will travel in a circular path of largest radius $r = 10 \text{ mm}$. The magnetic force evB on the electron provides centripetal acceleration i.e.,

$$\begin{aligned} evB &= mv^2/r, \quad \text{i.e.,} \\ mv &= eBr. \end{aligned} \quad (3)$$

Solve equations (1)–(3) for ϕ to get

$$\begin{aligned} \phi &= \frac{hc}{\lambda} - K_{\max} = \frac{hc}{\lambda} - \frac{m^2v^2}{2m} = \frac{hc}{\lambda} - \frac{e^2B^2r^2}{2m} \\ &= \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{6561 \times 10^{-10}} \\ &\quad - \frac{(1.6 \times 10^{-19})^2(3 \times 10^{-4})^2(10 \times 10^{-3})^2}{2(9.1 \times 10^{-31})} \\ &= 1.1 \text{ eV.} \end{aligned}$$

Q 73. Two metal plates are placed inside a vacuum tube. Electromagnetic radiations of frequency ν are incident on the left plate (work function $\phi < h\nu$). The fastest photoelectron travel from the point A to B in a straight line, pass through a hole at B, and then move in a circular arc from B to C. There is a uniform magnetic field \vec{B} in the region of curved path. Which of the following option(s) is (are) correct?



- (A) The time taken by the electron to travel from B to C is $\pi m / (2eB)$.
- (B) The radius of the circular path is $\sqrt{\frac{2m(h\nu - \phi)}{e^2 B^2}}$.
- (C) The radius of the circular path is $\sqrt{\frac{m(h\nu - \phi)}{2e^2 B^2}}$.
- (D) The direction of magnetic field is into the paper.

Sol. The energy of the fastest electron is equal to the maximum kinetic energy of the ejected photoelectrons

$$K_{\max} = h\nu - \phi.$$

Its speed is given by

$$v = \sqrt{\frac{2(h\nu - \phi)}{m}}.$$

The electron covers one quadrant of a circle (of radius r) from B to C. The magnetic force on an electron of charge $q = -e = -1.6 \times 10^{-19}$ C is $\vec{F} = -e\vec{v} \times \vec{B}$. The direction of \vec{F} is towards the centre of the circle. Hence, the direction of magnetic field \vec{B} should be into the paper.

The magnetic force $F = evB$ provides centripetal acceleration for circular motion of the electron i.e., $evB = mv^2/r$, which gives

$$r = \frac{mv}{eB} = \frac{m}{eB} \sqrt{\frac{2(h\nu - \phi)}{m}}.$$

The time taken by the electron to travel from B to C is

$$t = \frac{\pi r/2}{v} = \frac{\pi m}{2eB}.$$

Note that time is independent of the electron's velocity.

Ans. A, B, D \square

Q 74. Photoelectrons are emitted when 400 nm radiation is incident on a surface of work function 1.9 eV. These photoelectrons pass through a region containing α -particles. A maximum energy electron combines with an α -particle to form a He^+ ion, emitting a single photon in this process. He^+ ions thus formed are in their fourth excited state. Find the energies (in eV) of the photons lying in the 2 eV to 4 eV range, that are likely to be emitted during and after the combination. [Take $h = 4.14 \times 10^{-15}$ eV s.] (1999)

Sol. The energy of incident photon is

$$\frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9} \times 1.6 \times 10^{-19}} = 3.1 \text{ eV.}$$

In photoelectric effect, maximum kinetic energy of the ejected electron is

$$K_{\max} = hc/\lambda - \phi = 3.1 - 1.9 = 1.2 \text{ eV.}$$

The combination reaction is, $\text{He} + e \rightarrow \text{He}^+ + h\nu$, where $h\nu$ represents the emitted photon. The ion He^+ is hydrogen like with $Z = 2$. The principal quantum number in fourth excited state of He^+ is $n = 5$. The energy in n^{th} levels of He^+ is

$$E_n = -\frac{13.6Z^2}{n^2}. \quad (1)$$

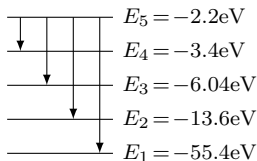
Substitute the values in equation (1) to get $E_1 = -54.4 \text{ eV}$, $E_2 = -13.6 \text{ eV}$, $E_3 = -6.04 \text{ eV}$, $E_4 = -3.4 \text{ eV}$ and $E_5 = -2.2 \text{ eV}$. Using energy conservation in combination reaction

$$E_{\text{He}} + E_e = E_{\text{He}^+} + E_{h\nu},$$

with $E_{\text{He}} = 0$, $E_e = K_{\max} = 1.2 \text{ eV}$, $E_{\text{He}^+} = E_5 = -2.2 \text{ eV}$, we get

$$E_{h\nu} = 1.2 - (-2.2) = 3.4 \text{ eV.}$$

After combination, He^+ can de-excite to the lower states by emitting photons as shown in the figure.



The energy of these transitions are $\Delta E_{5 \rightarrow 4} = E_5 - E_4 = 1.2\text{ eV}$, $\Delta E_{5 \rightarrow 3} = 3.84\text{ eV}$, $\Delta E_{5 \rightarrow 2} = 11.4\text{ eV}$, and $\Delta E_{5 \rightarrow 1} = 53.2\text{ eV}$, out of which $\Delta E_{5 \rightarrow 3} = 3.84\text{ eV}$ lies in the range of 2 eV to 4 eV . There is a possibility that He^+ makes multiple transitions during the de-excitation i.e., it can make transition $5 \rightarrow 4$ and then $4 \rightarrow 3$ or $4 \rightarrow 2$ or $4 \rightarrow 1$ and so on. We encourage you to verify that only $\Delta E_{4 \rightarrow 3} = E_4 - E_3 = 2.64\text{ eV}$ is less than 4 eV .

Ans. During combination 3.4 eV . After combination 3.84 eV , 2.64 eV . \square

Q 75. Light from discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV . The work function for sodium is 1.82 eV . Find, [Given, ionization potential of hydrogen is 13.6 eV .] (1992)

- the energy of the photons causing the photoelectrons emission.
- the quantum numbers of the two levels involved in the emission of these photons.
- the change in the angular momentum of the electron in the hydrogen atom in the above transition.

- (d) the recoil speed of the emitting atom assuming it to be at rest before the transition.

Sol. In photoelectric effect, the kinetic energy of the fastest photoelectrons is given by $K_{\max} = E - \phi$. Thus, the energy of incident photon is

$$E = K_{\max} + \phi = 0.73 + 1.82 = 2.55 \text{ eV}.$$

The photons are emitted when electron makes a transition between the two energy levels. The energy of n^{th} level in hydrogen atom is

$$E_n = -\frac{13.6}{n^2} \text{ eV}. \quad (1)$$

Substitute the values of n in equation (1) to get $E_1 = -13.6 \text{ eV}$, $E_2 = -3.4 \text{ eV}$, $E_3 = -1.5 \text{ eV}$, $E_4 = -0.85 \text{ eV}$, etc. The energy gap between $n = 4$ and $n = 2$ is $\Delta E = E_4 - E_2 = 2.55 \text{ eV}$. Thus, the transition between these levels emits a photon of energy 2.55 eV .

The angular momentum of the electron in n^{th} energy level is

$$L_n = nh/(2\pi).$$

Thus, the change in angular momentum for $n = 4$ to $n = 2$ transition is $\Delta L = L_2 - L_4 = -h/\pi$.

The linear momentum of the atom-photon system is conserved. Initially, linear momentum is zero i.e., $p_i = 0$. Finally, linear momentum of the photon is E/c , linear momentum of the hydrogen atom is mv , and of

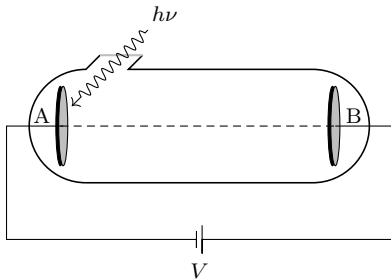
the total system is $p_f = E/c + mv$. Using, $p_i = p_f$, we get

$$v = -\frac{E}{mc} = -\frac{2.55 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27} \times 3 \times 10^8} = -0.814 \text{ m/s}, \quad (\text{recoil}).$$

Ans. (a) 2.55 eV (b) $4 \rightarrow 2$ (c) $-\frac{h}{\pi}$ (d) 0.814 m/s

□

Q 76. Two metal plates are placed inside a vacuum tube. Electromagnetic radiations of frequency ν are incident on the left plate (work function $\phi < h\nu$). A power supply maintains a potential difference V between two plates and a photocurrent i flows in the circuit. Assume that photoelectrons move parallel to the line AB and the plates are perpendicular to AB. The force F applied by the incident photoelectrons on the plate B satisfies



(A) $F = \frac{i}{e} [2m(h\nu - \phi + eV)]^{1/2}$

$$(B) F > \frac{i}{e} [2m(h\nu - \phi + eV)]^{1/2}$$

$$(C) F < \frac{i}{e} [2m(h\nu - \phi + eV)]^{1/2}$$

$$(D) F = \frac{i}{e} [2m(h\nu - \phi)]^{1/2}$$

Sol. The number of photoelectrons striking the plate B per second is

$$n = i/e.$$

The maximum kinetic energy of the ejected photoelectrons is

$$K_{\max} = h\nu - \phi.$$

All photoelectrons are not ejected with the maximum kinetic energy. They have a distribution of energy varying from zero to K_{\max} . Let K be kinetic energy of a photoelectron at the plate A. After ejection, the electron is accelerated by a potential difference V . Thus, kinetic energy of the electron at B is

$$K_B = K + eV.$$

The momentum of the photoelectron just before striking the plate B is

$$\begin{aligned} p_B &= \sqrt{2mK_B} = \sqrt{2m(K + eV)} \\ &< \sqrt{2m(K_{\max} + eV)} \\ &= \sqrt{2m(h\nu - \phi + eV)}. \end{aligned}$$

Thus, the force on the plate B due to incident photoelectrons is

$$F = np_B < \frac{i}{e} \sqrt{2m(h\nu - \phi + eV)}.$$

Ans. C \square

Q 77. There is a vacuum photocell whose one electrode is made of cesium ($\phi_{\text{Cs}} = 1.89$ eV) and the other of copper ($\phi_{\text{Cu}} = 4.47$ eV). The electrodes are shorted outside the cell. The maximum velocity of photoelectrons approaching the copper electrode when the cesium electrode is subjected to radiation of wavelength 220 nm is

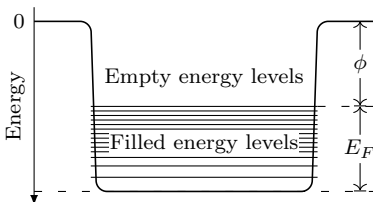
- (A) 6.4×10^5 m/s (B) 3.2×10^6 m/s
 (C) 1.14×10^6 m/s (D) 1.14×10^5 m/s

Sol. The maximum kinetic energy of photoelectrons ejected from the cesium electrode (emitter) when illuminated with radiations of wavelength $\lambda = 220$ nm is

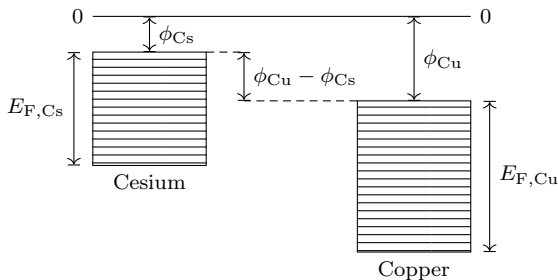
$$\begin{aligned} K_{\text{max}}^{\text{Cs}} &= hc/\lambda - \phi_{\text{Cs}} = 1240/220 - 1.89 \\ &= 5.63 - 1.89 = 3.74 \text{ eV.} \end{aligned}$$

The ejected electrons move towards the collector. The emitter (made of cesium) and the collector (made of copper) are shorted from outside the cell. For electrodes made up of dissimilar metals, shorting creates a potential difference between them. This potential difference is known as *contact potential*. To understand contact potential, we need to use a few concepts from solid state physics.

The valence electrons in metals are free to move but they cannot come out of it. They lie in a potential well. The potential well has closely spaced energy levels (band). The electrons are filled in these energy levels. The highest filled energy level is called *Fermi level*. The energy required to remove an electron from the Fermi level is equal to the work function ϕ . The depth of the well is equal to $\phi + E_F$, where E_F is called Fermi energy.

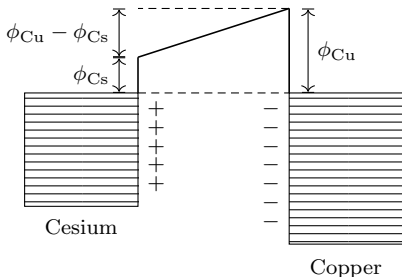


The work function and Fermi energy of dis-similar metals are different. For example, work function for cesium is $\phi_{\text{Cs}} = 1.89 \text{ eV}$ but that for copper is $\phi_{\text{Cu}} = 4.47 \text{ eV}$. Fermi level of cesium is 1.89 eV below the zero energy reference and Fermi level of copper is 4.47 eV below the zero energy reference.



When these metals come into contact, the electrons in high energy levels of cesium find lower energy levels of copper empty. Thus, electrons start moving from cesium to copper. The filled energy levels in cesium start depleting from the top and empty energy levels of copper start filling from the top. This process continues till the highest filled energy level of cesium and copper have the same energy i.e., both metals have the same Fermi level. This movement of electrons from cesium to copper leads to a net positive charge on cesium and a negative charge on copper. These charges create a potential difference between cesium and copper, with cesium at higher potential and copper at a lower potential. The potential difference is related to the difference in work functions of the two metals by

$$V_{Cs} - V_{Cu} = (\phi_{Cu} - \phi_{Cs})/e.$$



The photoelectrons are de-accelerated by the negative potential of copper. The loss in their kinetic energy is equal to $e(V_{Cs} - V_{Cu})$. The maximum kinetic energy of photoelectrons when they reach the copper electrode is

$$\begin{aligned}
 K_{\max}^{\text{Cu}} &= K_{\max}^{\text{Cs}} - e(V_{Cs} - V_{Cu}) \\
 &= hc/\lambda - \phi_{Cs} - (\phi_{Cu} - \phi_{Cs}) \\
 &= hc/\lambda - \phi_{Cu} = 5.63 - 4.47 = 1.16 \text{ eV}.
 \end{aligned}$$

The velocity of fastest photoelectrons at the copper electrode is

$$\begin{aligned}
 v &= \sqrt{\frac{2K_{\max}^{\text{Cu}}}{m}} = \sqrt{\frac{2(1.16)(1.6 \times 10^{-19})}{9.1 \times 10^{-31}}} \\
 &= 6.4 \times 10^5 \text{ m/s}.
 \end{aligned}$$

Ans. A \square

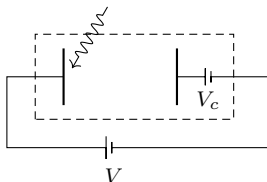
Q 78. A vacuum photocell of zinc electrode ($\phi = 3.74 \text{ eV}$) is subjected to electromagnetic radiation of

wavelength 262 nm. Its photoelectric current is cancelled if an external decelerating voltage 1.5 eV is applied. The *contact potential* of collector relative to the zinc electrode is

(A) 1 V (B) 0.5 V (C) 1.5 V (D) -1 V

Sol. The maximum kinetic energy of photoelectrons ejected from the zinc electrode when illuminated with radiations of wavelength $\lambda = 262$ nm is

$$\begin{aligned} K_{\max}^{\text{Zn}} &= hc/\lambda - \phi = 1240/262 - 3.74 \\ &= 4.73 - 3.74 = 0.99 \text{ eV} \approx 1 \text{ eV}. \end{aligned}$$



Let the collector has a positive contact potential V_c relative to the emitter. This positive potential will accelerate the electrons towards the collector. Thus, the maximum kinetic energy of the electrons when they reach the collector is

$$K_{\max}^{\text{C}} = K_{\max}^{\text{Zn}} + eV_c = (1 + V_c) \text{ eV}.$$

Hence, an external decelerating voltage $(1 + V_c)$ is required to stop the photocurrent. Its value is given as 1.5 V i.e., $1 + V_c = 1.5$, which gives $V_c = 0.5$ V.

The collector is at a positive contact potential $V_c = 0.5$ V w.r.t. the emitter. Its polarity is opposite to the polarity of external applied voltage.

Ans. B \square

de Broglie Wavelength: Particle

Q 79. The de Broglie wavelength of a ball of mass 100 g moving with a speed 100 m/s is

- (A) 6.63 m (B) 6.63 Å
(C) 6.63 fm (D) 6.63×10^{-35} m

Sol. The de Broglie wavelength of a ball of mass $m = 100$ g = 0.1 kg moving at a speed $v = 100$ m/s is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{(0.1)(100)} = 6.63 \times 10^{-35} \text{ m.}$$

The wavelength associated with ordinary object (like the ball here) is extremely small. These low wavelengths are difficult to observe. The momentum of an object need to be very small for its wavelength to be observable. The extremely small value of Planck's constant makes ordinary objects to look like particles rather than waves.

Ans. D \square

Q 80. If the kinetic energy of a free electron doubles, its de Broglie wavelength changes by the factor

(2005 m)

- (A) 1/2 (B) 2 (C) $1/\sqrt{2}$ (D) $\sqrt{2}$

Sol. The de Broglie wavelength of a particle of mass m moving with a momentum p (kinetic energy $K = p^2/(2m)$) is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}.$$

Thus, increasing the kinetic energy from K to $K' = 2K$ will decrease the wavelength from λ to $\lambda' = \lambda/\sqrt{2}$.

Exercise: Calculate de Broglie wavelength of an electron of kinetic energy $K = 100$ eV? *Ans.* 1.2 \AA .

Ans. C \square

Q 81. A particle moving with kinetic energy K has de Broglie wavelength λ . If energy ΔK is added to its kinetic energy, the wavelength become $\lambda/2$. Value of ΔK is (2020 m)

(A) $2K$ (B) $4K$ (C) $3K$ (D) K

Sol. The de Broglie wavelength of the particle is λ when its kinetic energy is K . The wavelength is reduced to $\lambda' = \lambda/2$ when its kinetic energy is increased to $K' = K + \Delta K$. Substitute these values in de Broglie wavelength expression $\lambda = h/\sqrt{2mK}$ to get

$$\lambda = \frac{h}{\sqrt{2mK}}, \quad (1)$$

$$\frac{\lambda}{2} = \frac{h}{\sqrt{2m(K + \Delta K)}}, \quad (2)$$

Divide equation (1) by (2) and solve to get $\Delta K = 3K$. The kinetic energy of a particle need to be increased four times to reduce its wavelength by half.

Exercise: What amount of energy should be added to an electron to reduce its de Broglie wavelength from 1 \AA to 0.5 \AA .

Ans. C \square

Q 82. An electron of mass m_e and a proton of mass m_p are accelerated by the same potential difference. The ratio of the de Broglie wavelength associated with an electron to that associated with a proton is
(A) 1 (B) m_p/m_e (C) m_e/m_p (D) $\sqrt{m_p/m_e}$

Sol. The charge on an electron and a proton are of the equal magnitudes. Thus, when an electron and a proton are accelerated by the same potential, both gains equal amount of kinetic energy (say K). The de Broglie wavelength of these particles are

$$\lambda_e = h/\sqrt{2m_e K},$$

$$\lambda_p = h/\sqrt{2m_p K}.$$

Thus, $\lambda_e/\lambda_p = \sqrt{m_p/m_e} = \sqrt{1836} = 43$. For equal kinetic energies, a lighter particle has larger wavelength.

Ans. D \square

Q 83. An α -particle and a proton are accelerated from rest by a potential of 100 V . After this, their de-Broglie wavelengths are λ_α and λ_p respectively. The ratio λ_p/λ_α , to the nearest integer, is (2010)

Sol. The de Broglie wavelength of a particle with momentum p is given by

$$\lambda = h/p.$$

The momentum and kinetic energy of a particle of mass m are related by

$$p = \sqrt{2mK}.$$

The kinetic energy of a charge q , accelerated through potential V , is given by $K = qV$. Thus,

$$\lambda = h/\sqrt{2mK} = h/\sqrt{2mqV},$$

which gives

$$\begin{aligned} \frac{\lambda_p}{\lambda_\alpha} &= \sqrt{\frac{2m_\alpha q_\alpha V}{2m_p q_p V}} = \sqrt{\frac{2 \cdot 4u \cdot 2e \cdot 100}{2 \cdot 1u \cdot 1e \cdot 100}} \\ &= \sqrt{8} = 2.8 \approx 3. \end{aligned}$$

Ans. 3 \square

Q 84. The potential energy of a particle varies as,

$$U(x) = \begin{cases} E_0 & \text{for } 0 \leq x \leq 1, \\ 0 & \text{for } x > 1. \end{cases}$$

For $0 \leq x \leq 1$, the de-Broglie wavelength is λ_1 and for $x > 1$, the de-Broglie wavelength is λ_2 . Total energy of the particle is $2E_0$. Find λ_1/λ_2 . (2005)

Sol. The de-Broglie wavelength of a particle of mass m , momentum p and kinetic energy K is given by

$$\lambda = h/p = h/\sqrt{2mK}.$$

The total energy is $U + K = 2E_0$. The kinetic energy and de-Broglie wavelength of the particle in the region $0 \leq x \leq 1$ are

$$\begin{aligned} K_1 &= 2E_0 - U = 2E_0 - E_0 = E_0, \\ \lambda_1 &= h/\sqrt{2mE_0}, \end{aligned} \quad (1)$$

and in the region $x > 1$ are

$$\begin{aligned} K_2 &= 2E_0 - U = 2E_0 - 0 = 2E_0, \\ \lambda_2 &= h/\sqrt{4mE_0}. \end{aligned} \quad (2)$$

Divide equation (1) by (2) to get $\lambda_1/\lambda_2 = \sqrt{2}$.

Ans. $\sqrt{2}$ \square

Q 85. An electron of mass m and initial velocity $\vec{v}_0 = v_0\hat{i} + v_0\hat{j}$ is placed in an electric field $\vec{E} = -E_0\hat{k}$. If λ_0 is initial de-Broglie wavelength of electron, its de-Broglie wavelength at time t is given by (2020 m)

$$\begin{aligned} \text{(A)} \quad & \lambda_0/\sqrt{1 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}} & \text{(B)} \quad & \lambda_0/\sqrt{2 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}} \\ \text{(C)} \quad & \lambda_0/\sqrt{1 + \frac{e^2 E_0^2 t^2}{2m^2 v_0^2}} & \text{(D)} \quad & \lambda_0\sqrt{2}/\sqrt{1 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}} \end{aligned}$$

Sol. Initial velocity of the electron is $\vec{v}_0 = v_0\hat{i} + v_0\hat{j}$, its momentum is $\vec{p}_0 = mv_0\hat{i} + mv_0\hat{j}$ and its de Broglie wavelength is

$$\lambda_0 = \frac{h}{|\vec{p}_0|} = \frac{h}{\sqrt{2}mv_0}. \quad (1)$$

The force on an electron of charge $q = -e$ in an electric field $\vec{E} = -E_0\hat{k}$ is $\vec{F} = q\vec{E} = eE_0\hat{k}$. The acceleration of

the electron is

$$\vec{a} = \frac{\vec{F}}{m} = \frac{eE_0}{m} \hat{k}.$$

The electron moves with a constant acceleration \vec{a} . Its velocity at time t is given by

$$\vec{v} = \vec{v}_0 + \vec{a}t = v_0\hat{i} + v_0\hat{j} + \frac{eE_0t}{m} \hat{k}.$$

The magnitude of its momentum at time t is

$$|\vec{p}| = m|\vec{v}| = \sqrt{2}mv_0\sqrt{1 + \frac{e^2E_0^2t^2}{2m^2v_0^2}}.$$

The de Broglie wavelength of the electron at time t is

$$\begin{aligned} \lambda &= \frac{h}{|\vec{p}|} = \frac{h}{\sqrt{2}mv_0\sqrt{1 + \frac{e^2E_0^2t^2}{2m^2v_0^2}}} \\ &= \frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{2m^2v_0^2}}} \quad (\text{using (1)}) \end{aligned}$$

Ans. C \square

Q 86. An electron of mass m and magnitude of charge $|e|$ initially at rest gets accelerated by a constant electric field E . The rate of change of de-Broglie wavelength of this electron at time t is [ignore relativistic effects]

(2020 m)

(A) $-\frac{h}{|e|Et^2}$ (B) $\frac{|e|Et}{h}$ (C) $-\frac{h}{|e|Et}$ (D) $-\frac{h}{|e|E\sqrt{t}}$

Sol. The force on an electron moving in an electric field E is $F = |e|E$. Its acceleration is $a = |e|E/m$ and its velocity at time t is

$$v = u + at = 0 + |e|Et/m = |e|Et/m.$$

The de Broglie wavelength of the electron at time t is

$$\lambda = \frac{h}{mv} = \frac{h}{|e|Et}.$$

The rate of change of de Broglie wavelength with time t is

$$\frac{d\lambda}{dt} = -\frac{h}{|e|Et^2}.$$

We encourage you to plot time-wavelength (t - λ) graph. Notice how wavelength varies with time when an electron is accelerated in a uniform electric field. When the electron speed increases close to the speed of light, the non-relativistic relation between momentum and energy gives erroneous results.

Ans. A \square

Q 87. A particle is moving 5 times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is 1.878×10^{-4} . The mass of the particle is close to (2020 m)

- (A) 9.1×10^{-31} kg (B) 9.7×10^{-28} kg
(C) 1.2×10^{-28} kg (D) 4.8×10^{-27} kg

Sol. Let m_e be the mass and v_e be the velocity of the electron. The de Broglie wavelength of the electron is given by

$$\lambda_e = \frac{h}{m_e v_e}.$$

Let m_p be mass of the particle. The velocity of the particle is $v_p = 5v_e$ and its wavelength is

$$\lambda_p = \frac{h}{m_p v_p} = \frac{h}{5m_p v_e}.$$

The ratio λ_p/λ_e is

$$\frac{\lambda_p}{\lambda_e} = \frac{m_e}{5m_p} = 1.878 \times 10^{-4}.$$

Substitute $m_e = 9.1 \times 10^{-31}$ kg in the above equation to get $m_p = 9.7 \times 10^{-28}$ kg.

Ans. B \square

Q 88. A thermal neutron has a kinetic energy kT where $T = 300$ K is the room temperature. Such neutrons are in thermal equilibrium with normal surroundings. Which of the option(s) is (are) correct?

- (A) The kinetic energy of thermal neutron is 25 meV.
- (B) The speed of the thermal neutrons is of the order of 2000 m/s.
- (C) de Broglie wavelength of thermal neutron is 1.8 Å.
- (D) Thermal neutrons are used to study the atomic structure of materials.

Sol. The kinetic energy of a thermal neutron is

$$\begin{aligned} K &= kT = (1.38 \times 10^{-23})(300) \text{ J} = 4.14 \times 10^{-21} \text{ J} \\ &= \frac{4.14 \times 10^{-21}}{1.6 \times 10^{-19}} \text{ eV} = 0.025 \text{ eV}. \end{aligned}$$

Note that the kinetic energy of a thermal neutron is of the order of 25 meV. The speed of thermal neutrons have a Maxwellian distribution. The speed corresponding to the kinetic energy kT is

$$\begin{aligned} v &= \left(\frac{2K}{m_n} \right)^{1/2} = \left(\frac{2 \times 4.14 \times 10^{-21}}{1.67 \times 10^{-27}} \right)^{1/2} \\ &= 2200 \text{ m/s}. \end{aligned}$$

The de Broglie wavelength of the thermal neutron is

$$\begin{aligned} \lambda &= \frac{h}{\sqrt{2m_n K}} = \frac{6.63 \times 10^{-34}}{\sqrt{2(1.6 \times 10^{-27})(4.14 \times 10^{-21})}} \\ &= 1.8 \times 10^{-10} \text{ m} = 1.8 \text{ \AA}. \end{aligned}$$

Note that de Broglie wavelength of thermal neutrons is of the order of atomic separations in solid materials. The thermal neutrons will undergo diffraction when passing through the material. These diffraction patterns are used to study the atomic structure of the material.

Ans. A, B, C, D \square

Q 89. A particle is moving with a speed much smaller than the speed of light. If the speed of the particle is increased by 1%, its de Broglie wavelength will

- (A) decrease by 1% (B) increase by 1%
(C) decrease by 2% (D) increase by 2%

Sol. The de Broglie wavelength of a particle of mass m moving with a speed v is given by

$$\lambda = \frac{h}{mv}.$$

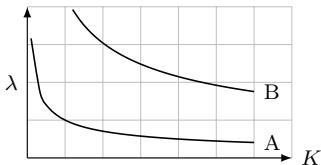
The speed of the particle is increased by 1% to a new value $v' = v + 0.01v$. The new wavelength is

$$\begin{aligned}\lambda' &= \frac{h}{mv'} = \frac{h}{mv(1 + 0.01)} = \frac{h}{mv}(1 + 0.01)^{-1} \\ &\approx \lambda(1 - 0.01) = 0.99\lambda.\end{aligned}$$

Thus, de Broglie wavelength decreased by 1% when speed of the particle is increased by 1%. We encourage you to derive the expression $\Delta\lambda/\lambda = -\Delta v/v$.

Ans. A \square

Q 90. The figure shows variation of de Broglie wavelength λ with the kinetic energy K for two particle - an electron and a proton. Which of the following option(s) is (are) correct?



- (A) Curve A is for an electron and B is for a proton
(B) Curve A is for a proton and B is for an electron
(C) For equal K , the wavelength of a lighter particle is greater than that of a heavier particle.
(D) For equal K , the wavelength of a heavier particle is greater than that of a lighter particle.

Sol. The de Broglie wavelength of an electron of mass m_e and kinetic energy K is given by

$$\lambda_e = \frac{h}{\sqrt{2m_e K}}.$$

The de Broglie wavelength of a proton of mass m_p and kinetic energy K is given by

$$\lambda_p = \frac{h}{\sqrt{2m_p K}}.$$

The ratio $\lambda_e/\lambda_p = \sqrt{m_p/m_e} = \sqrt{1836} \approx 43$ is a constant. For equal kinetic energy K , the wavelength of a lighter particle is greater than that of a heavier particle.

Exercise: Calculate the de Broglie wavelengths of an electron and proton moving with kinetic energy 1.0 eV. At what value of kinetic energy will their wavelengths be equal to 1.0 Å.

Ans. B, C □

Q 91. An electron is moving with a momentum 1.0 keV/c. The amount of energy to be added to the electron to make its wavelength equal to 0.50 Å is

- (A) 0.38 keV (B) 0.6 keV
(C) 1.2 keV (D) 0.38 MeV

Sol. The momentum of an electron of de Broglie wavelength $\lambda = 0.50 \text{ \AA}$ is

$$p = h/\lambda,$$

and its kinetic energy (using non-relativistic formula) is

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}.$$

The kinetic energy of an electron having momentum $p_0 = 1.0 \text{ keV}/c$ is

$$K_0 = \frac{p_0^2}{2m}.$$

Thus, energy that should be added to an electron of rest mass energy $mc^2 = 511 \text{ keV}$ is

$$\begin{aligned}\Delta K &= K - K_0 = \frac{h^2}{2m\lambda^2} - \frac{p_0^2}{2m} \\ &= \frac{1}{2mc^2} \left(\frac{hc}{\lambda} \right)^2 - \frac{(p_0c)^2}{2mc^2} \\ &= \frac{1}{2(511)} \left(\frac{12.4 \text{ keV} \cdot \text{\AA}}{0.5 \text{ \AA}} \right)^2 - \frac{(1)^2}{2(511)} \\ &= 0.6 \text{ keV}.\end{aligned}$$

We encourage you solve the problem using relativistic formula for energy $E^2 = p^2c^2 + m^2c^4$.

Ans. B (or A) \square

de Broglie Wavelength: Electron-Photon

Q 92. A potential difference V is applied to an X-ray tube. The de Broglie wavelength of an electron of mass m accelerated in this X-ray tube is λ_e and the minimum wavelength of generated X-ray spectrum is λ_p . If V is in volt and wavelengths are in \AA then which of the following option(s) is (are) correct?

- (A) $\lambda_e = 12.27/\sqrt{V}$ (B) $\lambda_p = 12450/V$
 (C) $\lambda_e/\lambda_p \propto \sqrt{V}$ (D) $\lambda_e/\lambda_p \propto V$

Sol. The kinetic energy of an electron accelerated through a potential V is $K = eV$. Its de Broglie wavelength is given by

$$\lambda_e = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}} = \frac{12.27}{\sqrt{V}} \text{\AA}. \quad (1)$$

The photons of cut-off wavelength in X-ray spectrum have same energy as the energy of electron i.e., $E = hc/\lambda_p = eV$, which gives

$$\lambda_p = \frac{h}{p} = \frac{hc}{E} = \frac{hc}{eV} = \frac{12450}{V} \text{\AA}. \quad (2)$$

Note that the energy E and the momentum p of a photon are related by $E = pc$. Divide equation (1) by (2) to get

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{eV}{2mc^2}} = \frac{\sqrt{V}}{1012} \text{\AA}.$$

A typical voltage of 25 kV is applied in X-ray tube. For this voltage, we get $\lambda_e = 0.08 \text{\AA}$, $\lambda_p = 0.5 \text{\AA}$ and $\lambda_e/\lambda_p = 0.15$.

Ans. A, B, C \square

Q 93. An electron (of mass m) and a photon have the same energy E in the range of a few eV. The ratio of the de Broglie wavelength associated with the electron and the wavelength of the photon is (2020 m)

- (A) $\left(\frac{E}{2m}\right)^{1/2}$ (B) $\frac{1}{c}\left(\frac{2E}{m}\right)^{1/2}$
(C) $c(2mE)^{1/2}$ (D) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$

Sol. The de Broglie wavelength of an electron of energy E is given by $\lambda_e = h/\sqrt{2mE}$ and the wavelength of a photon of the same energy is $\lambda_p = hc/E$. The ratio of these two wavelengths is

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{E}{2mc^2}}.$$

From the above equation, de Broglie wavelength of an electron of energy E is equal to the wavelength of a photon of the same energy if $E \approx 1$ MeV. Note that $mc^2 = 0.511$ MeV.

The question state that the energy E is in the range of a few eV. This statement is not redundant. The defining equation for de Broglie wavelength is $\lambda = h/p$. The momentum of a particle moving with a non-relativistic speed is related to its energy E by $p = \sqrt{2mE}$. This relation is not true for particles moving with a relativistic speed i.e., a speed close to the speed of light. For example, $p = E/c$, for a photon moving with the speed of light.

Exercise 1: The de Broglie wavelength of an electron is the same as the wavelength of a X-ray photon of energy

50 keV. What is the kinetic energy of electron? *Ans.* 2.5 keV.

Exercise 2: Using non-relativistic formulas, calculate the speed and de Broglie wavelength of an electron of energies (i) 1 eV (ii) 1 keV (iii) 1 MeV and (iv) 1 GeV.

Ans. D \square

Q 94. A proton has kinetic energy $E = 100$ keV which is equal to energy of a photon. Let λ_1 be the de-Broglie wavelength of the proton and λ_2 be the wavelength of the photon. The ratio λ_1/λ_2 is proportional to (2004)
(A) E^0 (B) $E^{1/2}$ (C) E^{-1} (D) E^{-2}

Sol. The de-Broglie wavelength of a particle of mass m , momentum p , and kinetic energy E is given by

$$\lambda_1 = h/p = h/\sqrt{2mE}. \quad (1)$$

The wavelength of a photon of energy E is given by

$$\lambda_2 = hc/E. \quad (2)$$

Divide equation (1) by (2) to get

$$\lambda_1/\lambda_2 = \sqrt{E/(2mc^2)}.$$

We encourage you to calculate λ_1 and λ_2 for the given energy.

Ans. B \square

Q 95. An electron and a photon each have a wavelength of 4 Å. Which of the following option(s) is (are) correct? (2020 m)

- (A) Kinetic energy of the electron is 9.4 eV
(B) Energy of the photon is 3100 eV
(C) Momentum of the electron is 1.6×10^{-24} kg m/s.
(D) Momentum of photon is 1.6×10^{-24} kg m/s.

Sol. The particles of same wavelength will have equal momentum. Thus, the momentum of an electron and a photon of same wavelength $\lambda = 4 \text{ \AA}$ is

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{4 \times 10^{-10}} = 1.66 \times 10^{-24} \text{ kg m/s.}$$

The energy of a photon of wavelength $\lambda = 4 \text{ \AA}$ is

$$E_p = \frac{hc}{\lambda} = \frac{12400 \text{ \AA} \cdot \text{eV}}{4 \text{ \AA}} = 3100 \text{ eV.}$$

The kinetic energy of an electron of de Broglie wavelength $\lambda = 4 \text{ \AA}$ is

$$\begin{aligned} E_e &= \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{(hc/\lambda)^2}{2mc^2} \\ &= \frac{(3100)^2}{2(0.511 \times 10^6)} = 9.4 \text{ eV.} \end{aligned}$$

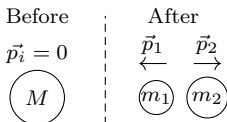
Note that the particles of same wavelength will have equal momentum but different energies.

Ans. A, B, C, D \square

de Broglie Wavelength: Collision

Q 96. A particle of mass M at rest decays into two particles of masses m_1 and m_2 having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles, λ_1/λ_2 is (1999)
 (A) m_1/m_2 (B) m_2/m_1 (C) 1 (D) $\sqrt{m_2}/\sqrt{m_1}$

Sol. The particle of mass M is at rest. Thus, the linear momentum of the system is zero before decay.



Let \vec{p}_1 and \vec{p}_2 be the linear momenta of the particles after decay. As there is no external force in decay process, the linear momentum of the system is conserved i.e.,

$$\vec{p}_1 + \vec{p}_2 = \vec{0},$$

which gives $|\vec{p}_1| = |\vec{p}_2|$. Hence, the de-Broglie wavelengths of two particles of mass m_1 and m_2 are equal i.e.,

$$\frac{\lambda_1}{\lambda_2} = \frac{h/|\vec{p}_1|}{h/|\vec{p}_2|} = 1.$$

Ans. C \square

Q 97. A nucleus A, with a finite de Broglie wavelength λ_A , undergoes spontaneous fission into two nuclei B

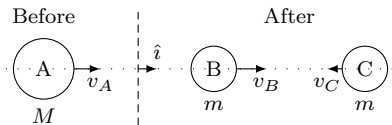
and C of equal mass. B flies in the same direction as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de Broglie wavelengths λ_B and λ_C of B and C are respectively

(2019 m)

- (A) $\lambda_A, 2\lambda_A$ (B) $2\lambda_A, \lambda_A$
 (C) $\lambda_A, \lambda_A/2$ (D) $\lambda_A/2, \lambda_A$

Sol. The linear momentum of nucleus A moving with de Broglie wavelength λ_A is

$$\vec{p}_A = (h/\lambda_A) \hat{i}.$$



The nuclei B and C are of equal mass m . Let v_B and v_C be the speeds of nuclei B and C. These speeds are related by $v_C = v_B/2$. The momentum of these nuclei are

$$\vec{p}_B = mv_B \hat{i},$$

$$\vec{p}_C = -mv_C \hat{i} = -(mv_B/2) \hat{i}.$$

The momentum of the system is conserved. The momentum before the fission, $\vec{p}_i = (h/\lambda_A) \hat{i}$, is equal to the momentum after the fission, $\vec{p}_f = \vec{p}_B + \vec{p}_C = mv_B/2 \hat{i}$. Solve $\vec{p}_i = \vec{p}_f$ to get

$$mv_B = 2h/\lambda_A.$$

Thus, momentum $p_B = mv_B = 2h/\lambda_A$ and $p_C = mv_B/2 = h/\lambda_A$. The de Broglie wavelengths of these nuclei are

$$\lambda_B = h/p_B = \lambda_A/2,$$

$$\lambda_C = h/p_C = \lambda_A.$$

Ans. D \square

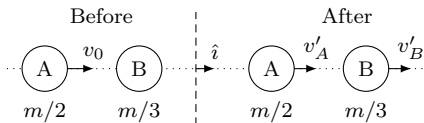
Q 98. Particle A of mass $m_A = m/2$ moving along the x -axis with velocity v_0 collides elastically with another particle B at rest having mass $m_B = m/3$. If both particles move along the x -axis after the collision, the change $\Delta\lambda$ in de-Broglie wavelength of particle A, in terms of its de-Broglie wavelength (λ_0) before collision is

(2020 m)

- (A) $\Delta\lambda = 5\lambda_0/2$ (B) $\Delta\lambda = 4\lambda_0$
 (C) $\Delta\lambda = 2\lambda_0$ (D) $\Delta\lambda = 3\lambda_0/2$

Sol. Let us consider particles A and B together as a system. The linear momentum of the system before the collision is

$$\vec{p}_i = \vec{p}_A + \vec{p}_B = (m/2)v_0 \hat{i} + \vec{0} = (mv_0/2) \hat{i}.$$



Let v'_A and v'_B be the speeds of A and B after the collision. The linear momentum of the system after the

collision is

$$\vec{p}_f = \vec{p}'_A + \vec{p}'_B = (m/2)v'_A \hat{i} + (m/3)v'_B \hat{i}.$$

The momentum of the system is conserved i.e., $\vec{p}_i = \vec{p}_f$, which gives

$$3v_0 = 3v'_A + 2v'_B. \quad (1)$$

The kinetic energy of the system is conserved in an elastic collision i.e.,

$$\begin{aligned} \frac{1}{2} \frac{m}{2} v_0^2 &= \frac{1}{2} \frac{m}{2} v'_A{}^2 + \frac{1}{2} \frac{m}{3} v'_B{}^2, \quad \text{i.e.,} \\ 3v_0^2 &= 3v'_A{}^2 + 2v'_B{}^2. \end{aligned} \quad (2)$$

Solve equations (1) and (2) to get $v'_A = v_0/5$ and $v'_B = 6v_0/5$.

The de Broglie wavelength of the particle A before the collision is

$$\lambda_A = \lambda_0 = \frac{h}{p_A} = \frac{h}{(m/2)v_0} = \frac{2h}{mv_0},$$

and that after collision is

$$\lambda'_A = \frac{h}{p'_A} = \frac{h}{(m/2)v'_A} = \frac{h}{(m/2)(v_0/5)} = \frac{10h}{mv_0} = 5\lambda_0.$$

The change in wavelength of the particle A is

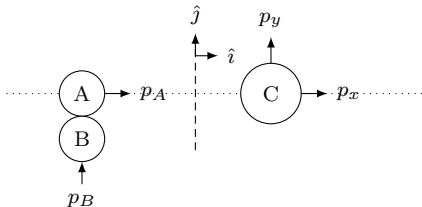
$$\Delta\lambda = \lambda'_A - \lambda_A = 5\lambda_0 - \lambda_0 = 4\lambda_0.$$

Q 99. Two particles A and B moves at right angle to each other. Their de Broglie wavelengths are λ_A and λ_B . The particles suffer perfectly inelastic collision. The de Broglie wavelength λ , of the final particle, is given by (2019 m)

- (A) $1/\lambda^2 = 1/\lambda_A^2 + 1/\lambda_B^2$ (B) $\lambda = \sqrt{\lambda_A \lambda_B}$
 (C) $\lambda = (\lambda_A + \lambda_B)/2$ (D) $2/\lambda = 1/\lambda_A + 1/\lambda_B$

Sol. Let us consider the particles A and B together as a system. The momentum of the system before the collision is

$$\vec{p}_i = \vec{p}_A + \vec{p}_B = (h/\lambda_A) \hat{i} + (h/\lambda_B) \hat{j}.$$



The particles combine together in a perfectly inelastic collision. The final momentum of the system is

$$\vec{p}_f = p_x \hat{i} + p_y \hat{j}.$$

The momentum of the system is conserved i.e., $\vec{p}_i = \vec{p}_f$, which gives $p_x = h/\lambda_A$ and $p_y = h/\lambda_B$. The momentum of the final particle is $p = \sqrt{p_x^2 + p_y^2}$. Substitute

expressions for momentum in terms of de Broglie wavelength to get

$$\frac{1}{\lambda^2} = \frac{p^2}{h^2} = \frac{p_x^2}{h^2} + \frac{p_y^2}{h^2} = \frac{1}{\lambda_A^2} + \frac{1}{\lambda_B^2}.$$

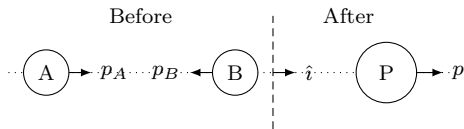
Ans. A \square

Q 100. A particle P is formed due to a completely inelastic collision of particles A and B having de Broglie wavelengths λ_A and λ_B respectively. If A and B were moving in opposite directions, then the de Broglie wavelength of P is (2019 m)

- (A) $\frac{\lambda_A \lambda_B}{\lambda_A + \lambda_B}$ (B) $\frac{\lambda_A \lambda_B}{|\lambda_A - \lambda_B|}$ (C) $\lambda_A - \lambda_B$ (D) $\lambda_A + \lambda_B$

Sol. Let us consider the particles A and B together as a system. The momentum of the system before the collision is

$$\vec{p}_i = \vec{p}_A + \vec{p}_B = (h/\lambda_A) \hat{i} - (h/\lambda_B) \hat{i}.$$



The particles A and B combine together in a perfectly inelastic collision. The momentum of combined particle will be towards \hat{i} if $p_A > p_B$ otherwise it will be towards $-\hat{i}$. The final momentum of the system when $p_A > p_B$ is

$$\vec{p}_f = p \hat{i},$$

where $p > 0$. The momentum of the system is conserved i.e., $\vec{p}_i = \vec{p}_f$, which gives

$$p = h/\lambda_A - h/\lambda_B.$$

Note that $\lambda_A < \lambda_B$ when $p_A > p_B$. The de Broglie wavelength of the particle P is

$$\lambda = \frac{h}{p} = \frac{h}{h/\lambda_A - h/\lambda_B} = \frac{\lambda_A \lambda_B}{\lambda_B - \lambda_A}.$$

The final momentum of the system when $p_A < p_B$ is

$$\vec{p}_f = -p\hat{i}.$$

The conservation of momentum, $\vec{p}_i = \vec{p}_f$, gives

$$p = (h/\lambda_B - h/\lambda_A).$$

Note that $\lambda_A > \lambda_B$ in this case. The de Broglie wavelength of the particle P is

$$\lambda = \frac{h}{p} = \frac{h}{h/\lambda_B - h/\lambda_A} = \frac{\lambda_A \lambda_B}{\lambda_A - \lambda_B}.$$

The results of above two cases can be combined as

$$\lambda = \frac{\lambda_A \lambda_B}{|\lambda_A - \lambda_B|} = \begin{cases} \frac{\lambda_A \lambda_B}{\lambda_A - \lambda_B}, & \text{if } \lambda_A > \lambda_B; \\ \frac{\lambda_A \lambda_B}{\lambda_B - \lambda_A}, & \text{if } \lambda_A < \lambda_B. \end{cases}$$

Ans. B \square

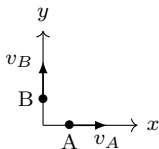
Q 101. Two identical non-relativistic particles A and B move at right angle to each other, having wavelengths λ_A and λ_B , respectively. The de Broglie wavelength of each particle in their centre of mass frame is

(A) $\lambda_A + \lambda_B$ (B) $\frac{\lambda_A \lambda_B}{\lambda_A + \lambda_B}$ (C) $\frac{\lambda_A \lambda_B}{\sqrt{\lambda_A^2 + \lambda_B^2}}$ (D) $\frac{2\lambda_A \lambda_B}{\sqrt{\lambda_A^2 + \lambda_B^2}}$

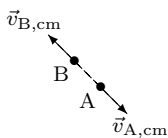
Sol. The momentum of particles A and B in the lab frame are given by

$$\vec{p}_A = (h/\lambda_A) \hat{i}, \quad \vec{p}_B = (h/\lambda_B) \hat{j}.$$

Lab frame



CoM frame



Let m_A and m_B be the masses of A and B. The velocities of these particles are

$$\vec{v}_A = \frac{h}{m_A \lambda_A} \hat{i}, \quad \vec{v}_B = \frac{h}{m_B \lambda_B} \hat{j}.$$

The velocity of the centre of mass is given by

$$\vec{v}_{\text{cm}} = \frac{m_A \vec{v}_A + m_B \vec{v}_B}{m_A + m_B}.$$

The velocities of A and B in the centre of mass frame are given by

$$\vec{v}_{A,\text{cm}} = \vec{v}_A - \vec{v}_{\text{cm}} = \frac{m_B}{m_A + m_B}(\vec{v}_A - \vec{v}_B),$$

$$\vec{v}_{B,\text{cm}} = \vec{v}_B - \vec{v}_{\text{cm}} = -\frac{m_A}{m_A + m_B}(\vec{v}_A - \vec{v}_B).$$

The momentum of A and B in the centre of mass frame are

$$\vec{p}_{A,\text{cm}} = m_A \vec{v}_{A,\text{cm}} = \frac{m_A m_B}{m_A + m_B}(\vec{v}_A - \vec{v}_B),$$

$$\vec{p}_{B,\text{cm}} = m_B \vec{v}_{B,\text{cm}} = -\frac{m_A m_B}{m_A + m_B}(\vec{v}_A - \vec{v}_B).$$

Note that the momentum of A and B in centre of mass frame are equal in magnitude but opposite in directions. The momentum of the system is zero in the centre of mass frame.

The de Broglie wavelength of two particles in the centre of mass frame are equal. These are given by

$$\lambda = \frac{h}{|\vec{p}_{A,\text{cm}}|} = \frac{(m_A + m_B)\lambda_A \lambda_B}{\sqrt{m_A^2 \lambda_A^2 + m_B^2 \lambda_B^2}}.$$

If particles are identical then $m_A = m_B$ and the wavelength is given by

$$\lambda = \frac{2\lambda_A \lambda_B}{\sqrt{\lambda_A^2 + \lambda_B^2}}.$$

Exercise: A neutron with kinetic energy $K = 25$ meV strikes a stationary deuteron. Find the de Broglie wavelengths of both particles in the centre of mass frame.

Ans. D \square

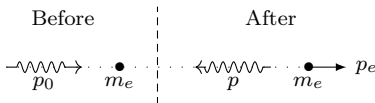
Q 102. Consider a collision between an X-ray photon of initial energy 50 keV and an electron at rest, in which the photon is scattered backward and the electron is knocked forward. The change in wavelength of the photon is

- (A) 0.024 Å (B) 0.048 Å (C) 0.511 Å (D) 0.248 Å

Sol. Before collision, the energy of striking photon is $E_0 = 50$ keV and its momentum is $p_0 = E_0/c$. The momentum of the electron is $p_{e0} = 0$ (because it is at rest). The collision of a photon with a free electron is a relativistic phenomenon. It is known as *Compton Effect*. We need to use relativistic relation between energy and momentum to get energy of the electron as

$$E_{e0} = \sqrt{p_{e0}^2 c^2 + (m_e c^2)^2} = m_e c^2.$$

The quantity $E_{e0} = m_e c^2 = 511$ keV is known as the rest mass energy of an electron.



After collision, let the momentum of backscattered photon be p and the momentum of knocked forward electron be p_e . The energy of the photon is $E = pc$ and that of the electron is (using relativistic relation)

$$E_e = \sqrt{p_e^2 c^2 + m_e^2 c^4}.$$

The conservation of momentum and energy gives

$$p_0 = E_0/c = p_e - p, \quad (1)$$

$$E_0 = pc + \sqrt{p_e^2 c^2 + m_e^2 c^4}. \quad (2)$$

Solve equations (1) and (2) to get

$$p = \frac{E_0 m_e c}{2E_0 + m_e c^2}.$$

The wavelength of photon before the collision is

$$\lambda_0 = \frac{h}{p_0} = \frac{hc}{E_0},$$

and the wavelength of backscattered photon is

$$\lambda = \frac{h}{p} = \frac{h(2E_0 + m_e c^2)}{E_0 m_e c}.$$

The increase in wavelength of the photon is given by

$$\begin{aligned} \Delta\lambda &= \lambda - \lambda_0 = \frac{2h}{m_e c} \\ &= \frac{2hc}{m_e c^2} = \frac{2 \times 12400 \text{ \AA} \cdot \text{eV}}{511 \times 10^3 \text{ eV}} = 2 \times 0.024 \text{ \AA}. \end{aligned}$$

Note that change in wavelength of a photon when it is backscattered from a free electron (at rest) is a constant. The quantity $h/m_e c$ is known as *Compton Wavelength* and its value for an electron is 0.024 \AA .

Ans. B \square

de Broglie Wavelength: Experiments

Q 103. A beam of electron is used in a Young's double slit experiment. The slit width is d . When the velocity of electron is increased, (2005)

- (A) no interference is observed
- (B) fringe width increases
- (C) fringe width decreases
- (D) fringe width remains same

Sol. The de-Broglie wavelength of an electron of mass m and velocity v is given by

$$\lambda = h/(mv). \quad (1)$$

The fringe width in Young's double slit experiment is related to wavelength λ and screen distance D by

$$\beta = \lambda D/d. \quad (2)$$

Eliminate λ from equations (1) and (2) to get

$$\beta = hD/(mvd).$$

Thus, both λ and β decrease when velocity v is increased.

Ans. C \square

Q 104. Consider the following statements:

Statement 1: Davisson-Germer experiment established the wave nature of electrons.

Statement 2: If electrons have wave nature, they can interfere and show diffraction.

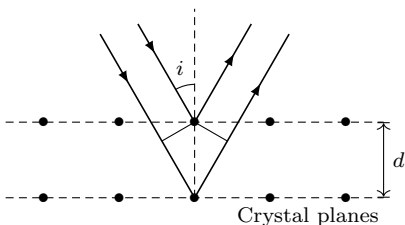
Which of the following option is correct? (2012 m)

- (A) Statement 1 is false, Statement 2 is true.
- (B) Statement 1 is true, Statement 2 is false.
- (C) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for statement 1.
- (D) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.

Sol. Both statements are true and statement 2 is the correct explanation for statement 1.

Ans. C \square

Q 105. Davisson and Germer demonstrated wave properties of electrons by diffracting them from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms interfere constructively.



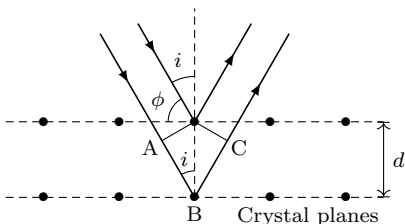
If a strong diffraction peak is observed when electrons are incident at an angle i from the normal to the

crystal planes with distance d between them, de Broglie wavelength λ of electrons can be calculated by the relationship [n is an integer] (2008 m)

- (A) $2d \cos i = n\lambda$ (B) $2d \sin i = n\lambda$
 (C) $d \cos i = n\lambda$ (D) $d \sin i = n\lambda$

Sol. The path difference between waves scattered from the successive planes is

$$\Delta x = AB + BC = d \cos i + d \cos i = 2d \cos i. \quad (1)$$



The path difference shall be an integer multiple of the de Broglie wavelength for constructive interference i.e.,

$$\Delta x = n\lambda, \quad (n \text{ is an integer}). \quad (2)$$

Eliminate Δx from equations (1) and (2) to get

$$2d \cos i = n\lambda.$$

Substituting $i = 90^\circ - \phi$ gives the well known *Bragg's* relation,

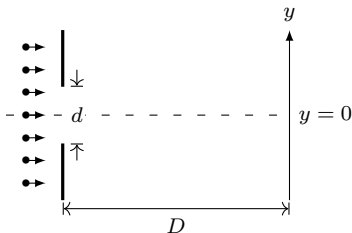
$$2d \sin \phi = n\lambda.$$

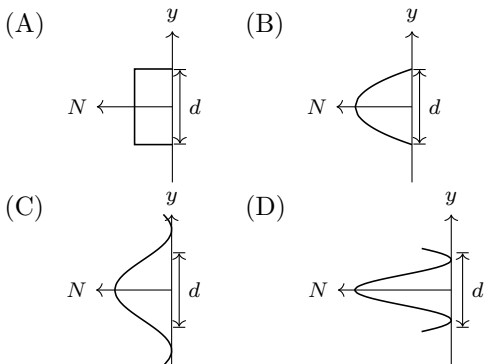
Exercise: The electrons accelerated by a potential V are diffracted from a crystal. If $d = 1 \text{ \AA}$ and $i = 30^\circ$, then V is equal to?

Ans. A \square

Q 106. In an experiment, electrons are made to pass through a narrow slit of width d comparable to their de Broglie wavelength. They are detected on a screen at a distance D from the slit (see figure). Which of the following graphs can be expected to represent the number of electrons N detected as a function of the detector position y ? [$y = 0$ corresponds to the middle of the slit]

(2008 m)





Sol. In this experiment, the electrons show their wave nature by undergoing diffraction through a single slit. The number of electrons N detected at position y of the screen is equivalent to the intensity at that point. Thus, the graph should be similar to intensity pattern in a single slit diffraction.

Options (A) and (B) don't represent diffraction pattern. Option (D) is excluded because width of the central maxima shall not be less than d .

Ans. C \square

Q 107. The mean separation between free electrons in a metal is approximately $l = 2 \text{ \AA}$. The metal is in thermal equilibrium at 27°C . The de Broglie wavelength of the electron is λ . Which of the following is correct?

(A) $\lambda \gg l$ (B) $\lambda \ll l$ (C) $\lambda \approx l$ (D) $\lambda = 31l$

Sol. The average kinetic energy of an electron when metal is in thermal equilibrium at an absolute temper-

ature T is

$$K = \frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT.$$

The momentum of the electron is given by

$$p = \sqrt{2mK} = \sqrt{3mkT}.$$

The de Broglie wavelength of the electron at temperature $T = 27^\circ\text{C} = 300\text{ K}$ is

$$\begin{aligned}\lambda &= \frac{h}{p} = \frac{h}{\sqrt{3mkT}} \\ &= \frac{6.63 \times 10^{-34}}{\sqrt{3(9.1 \times 10^{-31})(1.38 \times 10^{-23})(300)}} \\ &= 6.2 \times 10^{-9}\text{ m} = 62\text{ \AA}.\end{aligned}$$

The de Broglie wavelength of electrons is much larger than the mean separation between free electrons in a metal.

Exercise: Calculate the de Broglie wavelength of He atom in helium gas at STP and compare it with the mean separation between atoms.

Ans. D \square

Q 108. The diameter of a nucleus of an atom is of the order of 10^{-14} m . For an electron to be confined within a nucleus, its de Broglie wavelength would have to be less than or equal to the nucleus diameter. The kinetic energy of the electron for it to be confined in the nucleus would be

(A) 511 keV (B) 124 MeV (C) 15 GeV (D) 124 GeV

Sol. Let de Broglie wavelength of an electron confined in a nucleus is equal to the nuclear diameter i.e., $\lambda = 10^{-14}$ m. The momentum of the electron is

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{10^{-14}} = 6.63 \times 10^{-20} \text{ kg m/s.}$$

We need to use relativistic formula of energy if the value $pc \gg mc^2 = 0.511$ MeV. The value of pc is

$$\begin{aligned} pc &= (6.63 \times 10^{-20})(3 \times 10^8) = 1.989 \times 10^{-11} \text{ J} \\ &= 124.3 \text{ MeV.} \end{aligned}$$

Thus, $pc \gg mc^2$ and we need to use relativistic formula for the electron's energy

$$E = \sqrt{(pc)^2 + (mc^2)^2} \approx pc = 124.3 \text{ MeV.}$$

The kinetic energy of the electron is given by

$$K = E - mc^2 = 124.3 - 0.511 \approx 124 \text{ MeV.}$$

This is a huge energy. The electron cannot be confined within a nucleus. You can get the same result by using Heisenberg's uncertainty principle.

We encourage you to calculate kinetic energy of the electron using non-relativistic formula $K = p^2/(2m)$ and compare it with the value we obtained. Also, calculate the speed of the electron.

Ans. B \square

Q 109. The resolving power of an electron microscope operated at 16 kV is R . The resolving power of the electron microscope when operated at 4 kV is

(A) $2R$ (B) $\sqrt{2}R$ (C) $R/2$ (D) $R/\sqrt{2}$

Sol. The resolving power R of a microscope is inversely proportional to the wavelength λ . The de Broglie wavelength of an electron accelerated by a potential difference V is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} = \frac{12.27}{\sqrt{V}} \text{ \AA}.$$

Thus, resolving power of an electron microscope is proportional to the square root of the operating voltage V i.e.,

$$R = k\sqrt{V}, \quad (\text{for some constant } k).$$

Substitute given values to get $R = k\sqrt{16000}$ and

$$R' = k\sqrt{4000} = (1/2)k\sqrt{16000} = R/2.$$

Ans. C \square

Q 110. The smallest dimension (resolving power) that can be resolved by an electron microscope is equal to the de Broglie wavelength of its electron. What accelerating voltage would be required for the electrons to have the same resolving power as could be obtained using 100 keV gamma rays?

(A) 12400 V (B) 1240 V (C) 9791 V (D) 54 V

Sol. The de Broglie wavelength of the electron shall be equal to the wavelength of gamma rays for the electron microscope to have the same resolving power as

that of the gamma rays microscope. The wavelength of gamma-rays of energy $K = 100$ keV is

$$\lambda = \frac{hc}{E} = \frac{12400 \text{ \AA} \cdot \text{eV}}{100 \times 10^3 \text{ eV}} = 0.124 \text{ \AA}. \quad (1)$$

The wavelength of an electron accelerated by a potential difference V is given by

$$\lambda_e = \frac{h}{\sqrt{2meV}} = \frac{12.27}{\sqrt{V}} \text{ \AA}. \quad (2)$$

Substitute λ and λ_e from equations (1) and (2) in the equality $\lambda_e = \lambda$ to get the potential difference

$$V = \left(\frac{12.27}{0.124} \right)^2 = 9791 \text{ V}.$$

Which microscope i.e., the electron microscope or the gamma rays optical microscope seems more practical?

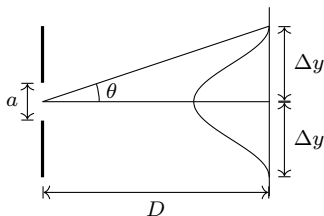
Ans. C \square

Q 111. A parallel stream of monoenergetic electrons falls normally on a single slit of width $a = 1.0 \mu\text{m}$. A screen is located at a distance $D = 50$ cm from the slit. The half-width of the central diffraction maximum formed on the screen is $\Delta y = 0.18$ mm. The speed of the incident electrons is

- (A) 1.01×10^5 m/s (B) 1.01×10^6 m/s
(C) 2.02×10^5 m/s (D) 2.02×10^6 m/s

Sol. The first minimum of a diffraction pattern by a single slit is obtained at a position given by

$$\lambda = a \sin \theta \approx a (\Delta y / D)$$



The momentum of an incident electron is related to its de Broglie wavelength by $p = mv = h/\lambda$. Thus, the velocity of the incident electrons is

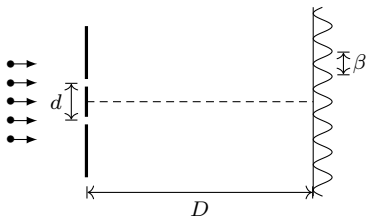
$$\begin{aligned}
 v &= \frac{h}{m\lambda} = \frac{hD}{ma\Delta y} \\
 &= \frac{(6.63 \times 10^{-34})(0.5)}{(9.1 \times 10^{-31})(10^{-6})(0.18 \times 10^{-3})} \\
 &= 2.02 \times 10^6 \text{ m/s.}
 \end{aligned}$$

Ans. D \square

Q 112. Two narrow slits are separated by a distance $d = 50 \mu\text{m}$. A screen is placed at a distance $D = 1.0 \text{ m}$ from the slits. A parallel beam of electrons accelerated by a potential difference $V = 25 \text{ volt}$ falls normally on the slits. The distance between two consecutive maximum of the pattern formed on the screen is

(A) $2.45 \mu\text{m}$ (B) $3.47 \mu\text{m}$ (C) $4.90 \mu\text{m}$ (D) $6.90 \mu\text{m}$

Sol. This is a Young's double slits experiment with a beam of electrons instead of the light.



The kinetic energy of an electron when it is accelerated by a potential difference V is $K = eV$. The de Broglie wavelength of incident electrons is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}.$$

The distance between consecutive maxima is equal to the fringe width of interference pattern. The fringe width is given by

$$\begin{aligned} \beta &= \frac{\lambda D}{d} = \frac{hD}{d\sqrt{2meV}} \\ &= \frac{(6.63 \times 10^{-34})(1.0)}{(50 \times 10^{-6})\sqrt{2(9.1 \times 10^{-31})(1.6 \times 10^{-19})(25)}} \\ &= 4.9 \times 10^{-6} \text{ m} = 4.9 \text{ } \mu\text{m}. \end{aligned}$$

Ans. C \square

Q 113. In the Davisson-Germer experiment, the electrons are accelerated by 54 V potential difference. These electrons are diffracted from a nickel crystal. The first maximum in the diffraction pattern was observed

at detector angle of $\theta = 50^\circ$. The lattice spacing d is related to the de Broglie wavelength of electron λ and the detector angle θ by Bragg's relation

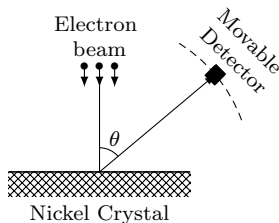
$$n\lambda = 2d \sin \phi = 2d \sin(90^\circ - \theta/2),$$

where $n = 1$ for the first maximum. The lattice spacing in the nickel crystal is?

(A) 1.65 Å (B) 0.91 Å (C) 0.45 Å (D) 1.23 Å

Sol. The de Broglie wavelength of an electron accelerated through a potential $V = 54$ volt is

$$\begin{aligned} \lambda &= \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}} \\ &= \frac{12.27}{\sqrt{V}} \text{ \AA} = \frac{12.27}{\sqrt{54}} \text{ \AA} = 1.66 \text{ \AA}. \end{aligned}$$



Substitute given values in Bragg's relation to get the lattice spacing in the nickel crystal as

$$d = \frac{\lambda}{2 \sin(90^\circ - \theta/2)} = \frac{1.66}{2 \sin 65^\circ} = 0.91 \text{ \AA}.$$

We don't stress on memorising numbers. The historical experiments like that of Davisson and Germer are a few exceptions.

Ans. B \square

de Broglie Wavelength: Miscellaneous

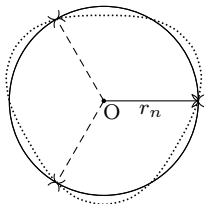
Q 114. Consider an electron in a hydrogen atom revolving in its second excited state (having radius 4.65 \AA). The de-Broglie wavelength of this electron is (2019 m)

(A) 3.5 \AA (B) 6.6 \AA (C) 12.9 \AA (D) 9.7 \AA

Sol. The perimeter of the n^{th} Bohr's orbit is n times the de-Broglie wavelength of the electron in that orbit (see figure) i.e.,

$$2\pi r_n = n\lambda, \quad (1)$$

where r_n is the radius of n^{th} Bohr's orbit.



In the second excited state of hydrogen atom, $n = 3$ and $r_n = 4.65 \text{ \AA}$. Substitute these values in equation (1) to get $\lambda = 9.7 \text{ \AA}$.

Ans. D \square

Q 115. A proton is fired from very far away towards a nucleus of charge $Q = 120e$, where e is electronic charge. It makes a closest approach of 10 fm to the nucleus. The de-Broglie wavelength (in units of fm) of proton at its start is [Take the proton mass $m_p = 5/3 \times 10^{-27}$ kg, $h/e = 4.2 \times 10^{-15}$ J s/C, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ m/F, 1 fm = 10^{-15} m.] (2012)

Sol. Let initial and final kinetic energies of the proton be K_i and K_f and corresponding potential energies be U_i and U_f . When proton is far away from the nucleus ($r \rightarrow \infty$), its potential energy is

$$U_i = \lim_{r \rightarrow \infty} \frac{1}{4\pi\epsilon_0} \frac{120e^2}{r} = 0.$$

At closest distance, the proton comes to rest momentarily, giving $K_f = 0$. The potential energy at closest distance is

$$U_f = \frac{1}{4\pi\epsilon_0} \frac{120e^2}{a},$$

where a is the distance of closest approach. Since electrostatic force is conservative, total energy is conserved i.e., $K_i + U_i = K_f + U_f$. Substitute the values to get

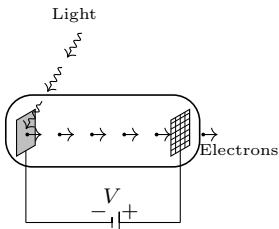
$$K_i = U_f = \frac{1}{4\pi\epsilon_0} \frac{120e^2}{a}.$$

The de-Broglie wavelength of the proton is given by

$$\begin{aligned}\lambda_i &= \frac{h}{p_i} = \frac{h}{\sqrt{2m_p K_i}} = \frac{h}{e} \sqrt{\frac{4\pi\epsilon_0 a}{240m_p}} \\ &= 4.2 \times 10^{-15} \left(\frac{10 \times 10^{-15}}{9 \times 10^9 \times 240 \times (5/3) \times 10^{-27}} \right)^{1/2} \\ &= 7 \text{ fm.}\end{aligned}$$

Ans. 7 \square

Q 116. Light of wavelength λ_{ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electron passing through the anode is λ_e , which of the following statement(s) is(are) true? (2016)



- (A) λ_e decreases with increase in ϕ and λ_{ph} .
 (B) λ_e is approximately halved, if d is doubled.

- (C) For large potential difference ($V \gg \phi/e$), λ_e is approximately halved if V is made four times.
- (D) λ_e increases at the same rate as λ_{ph} for $\lambda_{\text{ph}} < hc/\phi$.

Sol. In photo-electric effect, the maximum kinetic energy of the photo-electron ejected at the cathode, is given by

$$K_{\text{max,c}} = hc/\lambda_{\text{ph}} - \phi,$$

where λ_{ph} is the wavelength of the incident light and ϕ is the work function of the material. The ejected photo-electrons are accelerated from the cathode to the anode by a potential V . Thus, the maximum kinetic energy of the photo-electron at the anode is

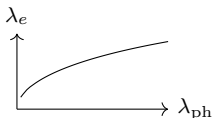
$$K_{\text{max,a}} = K_{\text{max,c}} + eV = hc/\lambda_{\text{ph}} - \phi + eV.$$

The linear momentum of the electron of mass m and kinetic energy K is given by $p = \sqrt{2mK}$. Thus, the minimum de-Broglie wavelength of the electron at the anode is

$$\begin{aligned}\lambda_e &= \frac{h}{p} = \frac{h}{\sqrt{2mK_{\text{max,a}}}} \\ &= \frac{h}{\sqrt{2m(hc/\lambda_{\text{ph}} - \phi + eV)}}.\end{aligned}\tag{1}$$

The wavelength λ_e increases with increase in ϕ and λ_{ph} as denominator of equation (1) decreases with increase in ϕ and λ_{ph} . Also, λ_e is independent of d . The order

of magnitudes of hc/λ_{ph} and ϕ is almost same. Thus, if $V \gg \phi/e$ then equation (1) reduces to $\lambda_e \approx h/\sqrt{2meV}$. Hence, λ_e is approximately halved if V is made four times.



The equation (1) is not linear in λ_{ph} and λ_e . Thus, the rates of increase of λ_e and λ_{ph} will be different. This can be verified by differentiating equation (1). You are encouraged to show that the slope $d\lambda_e/d\lambda_{\text{ph}}$ is different at different values of λ_{ph} .

Ans. C \square

Q 117. A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength λ ($\lambda < hc/\phi_0$). The fastest photoelectron has a de Broglie wavelength λ_d . A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in λ_d . Then the ratio $\Delta\lambda_d/\Delta\lambda$ is proportional to (2017)
 (A) λ_d^2/λ^2 (B) λ_d/λ (C) λ_d^3/λ (D) λ_d^3/λ^2

Sol. Here, λ is the wavelength of incident light and λ_d is the de Broglie wavelength of the fastest photoelectron. The fastest ejected photoelectron has the maximum kinetic energy which is given by

$$K_{\text{max}} = \frac{hc}{\lambda} - \phi_0. \quad (1)$$

The de Broglie wavelength of the photoelectron having kinetic energy $K_{\max} = \frac{p^2}{2m}$ is given by

$$\lambda_d = \frac{h}{p} = \frac{h}{\sqrt{2mK_{\max}}}, \quad (2)$$

where m is the mass of the electron and p is its linear momentum. Eliminate K_{\max} from equations (1) and (2) to get

$$\frac{h^2}{2m\lambda_d^2} = \frac{hc}{\lambda} - \phi_0. \quad (3)$$

Differentiate equation (3) to get

$$(-2) \frac{h^2}{2m\lambda_d^3} \Delta\lambda_d = (-1) \frac{hc}{\lambda^2} \Delta\lambda,$$

which gives

$$\frac{\Delta\lambda_d}{\Delta\lambda} = \frac{mc}{h} \frac{\lambda_d^3}{\lambda^2}.$$

Ans. (D) \square

Q 118. When photons of energy 4.25 eV strike the surface of a metal A , the ejected photoelectrons have maximum kinetic energy T_A eV and de-Broglie wavelength λ_A . The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50 \text{ eV})$. If the de-Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then,

(1994)

- (A) the work function of A is 2.25 eV.
(B) the work function of B is 4.20 eV.
(C) $T_A = 2.00$ eV.
(D) $T_B = 2.75$ eV.

Sol. In photoelectric effect, maximum kinetic energy of the ejected photoelectrons is given by $T = h\nu - \phi$. Thus,

$$T_A = 4.25 - \phi_A, \quad (1)$$

$$T_B = 4.70 - \phi_B. \quad (2)$$

Given,

$$T_B = T_A - 1.50, \quad (3)$$

$$\frac{\lambda_A}{\lambda_B} = \frac{h/\sqrt{2mT_A}}{h/\sqrt{2mT_B}} = \sqrt{\frac{T_B}{T_A}} = \frac{1}{2}. \quad (4)$$

Solve equations (3) and (4) to get $T_A = 2$ eV and $T_B = 0.5$ eV. Substitute T_A in equation (1) to get $\phi_A = 2.25$ eV and substitute T_B in equation (2) to get $\phi_B = 4.2$ eV.

Ans. A, B, C \square

Q 119. In a photoelectric effect set-up, a point source of light of power 3.2×10^{-3} W emits mono-energetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV and of radius 8.0×10^{-3} m. The efficiency of photoelectrons emission is one for every 10^6 incident photons. Assume that the

sphere is isolated and initially neutral and that photoelectrons are instantly swept away after emission.

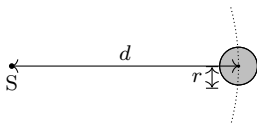
(1995)

- Calculate the number of photoelectrons emitted per second.
- Find the ratio of the wavelength of incident light to the de-Broglie wavelength of the fastest photoelectrons emitted.
- It is observed that the photoelectrons emission stops at a certain time t after the light source is switched on. Why?
- Evaluate the time t .

Sol. Let n be the number of photons emitted per second, each with energy $E_p = 5$ eV. The power of the source, 3.2×10^{-3} W, is equal to the energy of n photons i.e.,

$$n(5 \times 1.6 \times 10^{-19}) = 3.2 \times 10^{-3},$$

which gives $n = 4 \times 10^{15}$ photons/s.



The number of photons per unit area at a distance $d = 0.8$ m from the source is $n_d = \frac{n}{4\pi d^2}$. The photons lying in the area πr^2 strike the sphere. Thus, number

of photons striking per second on the sphere of radius $r = 8 \times 10^{-3}$ m is

$$\begin{aligned} n_s &= n_d(\pi r^2) = \frac{nr^2}{4d^2} = \frac{4 \times 10^{15} (8 \times 10^{-3})^2}{4 \times (0.8)^2} \\ &= 10^{11} \text{ photons/s.} \end{aligned}$$

Since one photoelectron is emitted for every 10^6 incident photons, the number of emitted photoelectrons per second is $n_s \times 10^{-6} = 10^5$ /s.

The wavelength of incident radiations is related to the photons energy by

$$hc/\lambda_p = E_p. \quad (1)$$

The kinetic energy of the fastest photoelectrons is given by

$$K_{\max} = h\nu - \phi = 5.0 - 3.0 = 2 \text{ eV.}$$

The de-Broglie wavelength of the fastest photoelectrons is

$$\lambda_e = h/p = h/\sqrt{2mK_{\max}}. \quad (2)$$

Divide equation (2) by (1) to get

$$\begin{aligned} \frac{\lambda_p}{\lambda_e} &= \frac{hc/E_p}{h/\sqrt{2mK_{\max}}} = \frac{c\sqrt{2mK_{\max}}}{E_p} \\ &= \frac{3 \times 10^8 \sqrt{2(9.1 \times 10^{-31})(2 \times 1.6 \times 10^{-19})}}{5 \times 1.6 \times 10^{-19}} \\ &= 286. \end{aligned} \quad (3)$$

The sphere becomes positively charged due to emission of photoelectrons. The emission stops when the potential on the sphere is equal to the stopping potential, which is 2 V. Let q be charge on the sphere when emission stops. Equating potential on the sphere to the stopping potential,

$$\frac{1}{4\pi\epsilon_0} \frac{q}{r} = 2,$$

we get $q = (4\pi\epsilon_0)2r = 1.78 \times 10^{-12}$ C. Number of photoelectrons emitted till the sphere accumulates charge q is

$$N = \frac{q}{e} = \frac{1.78 \times 10^{-12}}{1.6 \times 10^{-19}} = 1.11 \times 10^7.$$

Since the number of photoelectrons emitted per second is 10^5 /s, required time is $t = N/10^5 = 111$ s.

Ans. (a) 10^5 /s

(b) 286 (c) Emission of photoelectrons is stopped when its potential is equal to the stopping potential required for the fastest moving electrons. (d) 111 s \square

Q 120. X-rays of wavelength 1 \AA falls on photo-sensitive surface of a metal. The minimum de Broglie wavelength of ejected photoelectrons is

(A) 1.1 \AA (B) 0.11 \AA (C) 12.3 \AA (D) 1.23 \AA

Sol. The energy of X-ray photon of wavelength $\lambda = 1 \text{ \AA}$ is

$$E = \frac{hc}{\lambda} = \frac{12400 \text{ \AA} \cdot \text{eV}}{1 \text{ \AA}} = 12400 \text{ eV}.$$

The de Broglie wavelength is minimum for photoelectrons of the maximum kinetic energy. The maximum kinetic energy of photoelectrons ejected from a metal of work function ϕ is given by

$$K = E - \phi = 12400 - \phi \approx 12400 \text{ eV}.$$

Note that work function of metal is of the order of a few eV, which is negligible in comparison to the energy of X-ray photon. The de Broglie wavelength of an electron of kinetic energy K_{\max} is

$$\begin{aligned}\lambda_e &= \frac{h}{\sqrt{2mK_{\max}}} \\ &= \frac{6.63 \times 10^{-34}}{\sqrt{2(9.1 \times 10^{-31})(12400 \times 1.6 \times 10^{-19})}} \\ &= 0.11 \times 10^{-10} \text{ m} = 0.11 \text{ \AA}.\end{aligned}$$

Ans. B \square